Quantitative Geographical Analysis
Principal Components and Factor Analysis.
Background paper for March 19, 2001 seminar discussion.
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1. A Short Summary.

Principal components and factor analysis are widely used (and abused) techniques for understanding relations among many different variables. The techniques involve an extraordinarily rich menu of detailed choices and analytical procedures, but the general approach is simple: take a correlation matrix describing relations among a number of variables, and distill this matrix into a smaller number of dimensions that capture, as far as possible, the bulk of the information contained in the original dataset.

2. The 'Place' of PCA and Factor Analysis: the Case of Urban Geography.

Principal components analysis was devised in the early years of the twentieth century.[1] Factor analysis was developed around the same time, using similar mathematical procedures, in the field of educational psychology.[2] Educational researchers often encountered the problem of many different variables (say, scores on various tests, or grades in specific subjects) that were all measuring different aspects of the same underlying construct (aptitude, achievement, or, more controversially, innate intelligence). PCA and factor analysis provided valuable, but distinct solutions when analysts sought to examine the balance between general and unique aspects of complex, multivariate problems.

In the 1950s and 1960s, the development of the digital computer allowed the wider use of computationally-intensive techniques. PCA and factor analysis came to be used in a number of applications in the social and physical sciences, but in urban geography and urban sociology the trend accelerated at an especially rapid pace. One of the most important reasons for urban geographers' sudden infatuation with the approach was the pervasive influence of social theories from the Chicago School of Sociology; with newly-available computer power and the release of neighborhood-level information from the 1960 Census of Population and Housing, pca and factor analysis allowed analysts to flesh out the fascinating empirical details of theories that most scholars already understood and had accepted for years. The early 1960s, then, were a period when one of the binding constraints of scholarly inquiry was suddenly eliminated. To add force, many of the most talented young graduate students were trying to stake their reputations and establish their independence from the powerful old men of the discipline. The urban literature took a sudden and decisive turn towards quantification, measurement, and empirical investigations.

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[1] Pearson, K. (1901). "On Lines and Planes of Closest Fit to Systems of Points in Space." *Philosophical Magazine* 6(2), 559-572. See also Hotelling, H. (1933). "Analysis of a Complex of Statistical Variables into Principal Components." *Journal of Educational Psychology* 24, 417-441, 498-520.
[2] Spearman, C. (1904). "General Intelligence Objectively Determined and Measured." *American Journal of Psychology* 15, 201-293.

But the force of this disciplinary shift also generated a countervailing backlash a few years later. By the end of the 1970s the movement had run its course, and factor analysis and pca were not widely used in urban geography.[3] By the end of the 1980s, when urban geography was engaging new theories of economic restructuring, globalization, and widespread socio-cultural transformation, the analysis was often conducted in qualitative historical terms, or with the analytical arsenal of structuralist and poststructuralist theory. Let me be absolutely clear: the infusion of social theory into the urban literature was no less rigorous and valuable than the best multivariate statistics or geographic information science. Analysts have different skills and preferences. And there is also the question of matching theoretical questions to suitable analytical techniques: one wonders how to "model" the sea-change in cultural practices, consumption expectations, and political sensibilities after the end of a unified commitment to modernism in the early 1970s.

In any event, I saw an expanding area where factorial methods could shed important new light on an emerging debate. I became concerned with the proliferation of breathless, millenial writing on the changing nature of cities. By the middle of the 1990s, everyone seemed to have concluded that globalization had become so important that it had erased all that came before, and ushered in a completely new era for cities and urban life. From my perspective, all this apparent change had not really altered certain things about the metropolis: racial segregation and the spatial concentration of poverty had not disappeared; the division between singles and married couple-families with children in different neighborhoods seemed the same; and the effects of living in the suburbs on traffic congestion, spending decisions, and other aspects of life had not gone away. I set out to see what had changed, and what trends were simply new visible manifestations of the same fundamental processes. I turned to feminist urban research for theories of the relations between household arrangements and residential choice; and I looked to the factorial ecology tradition for my methods. It's a very strange mix: imagine a no-holds barred WWF event between Brian Berry and Linda McDowell. This is what I wrote to justify the synthesis:[4]

> Many theories have emerged to explain changes in urban form in the context of global, national, and regional economic transformation. Two literatures have been especially influential in shaping geographers' perceptions of contemporary urban change. In the global cities literature, authors emphasize how the rise of rapidly mobile transnational capital investment networks fuels changes in national and regional urban systems (Friedman, 1995; Sassen, 1994; Knox and Taylor, 1995). At an inter-urban scale, accelerated capital mobility and industrial change have ushered in repeated rounds of restructuring since the early 1970s. Many theorists also point to widening inequalities *within* cities, as growth in financial services inscribes an increasingly polarized occupational distribution atop eroding Fordist job structures rooted in unionized manufacturing. In a second literature, geographers concerned with a postmodern urbanism have sought to provide an alternative to still-hegemonic theories derived from the Chicago School (Scott and Soja, 1996; Knox, 1991, 1993; Dear and Flusty, 1998). Adopting an explicit focus on the internal urban expression of larger-scale postindustrial shifts, these analysts posit that the relations among industrial

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[3] For contributions and assessments in the urban factorial ecology literature, I recommend these sources: Berry, Brian J.L., and Horton, Frank E. (1970). *Geographic Perspectives on Urban Systems.* Englewood Cliffs, NJ: Prentice-Hall. Berry, Brian J.L., and Kasarda, John D. (1977). *Contemporary Urban Ecology*. New York: Macmillan. Davies, Wayne K.D. (1984). *Factorial Ecology*. Aldershot: Gower Press. Johnston, R.J. (1984). *City and Society: An Outline for Urban Geography*. London: Hutchinson. On the more general point of shifting questions and techniques in the urban literature, consult Livingstone's chapter in *The Geographical Tradition*, or, for the one of the most comprehensive urban bibliographies published in the last decade, see Harris, Chauncy D. 1995. "'The nature of cities' and urban geography in the last half century." *Urban Geography* 18(1): 15-35.

[4] Wyly, Elvin K. (1999). "Continuity and Change in the Restless Urban Landscape." *Economic Geography* 75(4), 309-338.

restructuring, sociocultural change, and the built environment have fundamentally altered the processes, patterns, and meanings of residential differentiation. These changes are held to be most pronounced in the emergence of new middle classes (Wright, 1989). The aesthetic sensibilities and consumption norms of these new classes demand new types of housing and amenities, as well as new retail, entertainment, and office spaces (Knox, 1991). Concomitant changes in the role of financial institutions, the state, and developers in providing these new elements of the urban landscape are said to undermine theories of residential mobility and neighborhood change (such as those presented in Clark and Dieleman, 1996 and Myers, 1990) that are rooted in ecological frameworks.

The emergence and growth of these literatures on global cities and postmodern urbanism signify the culmination of two broader disciplinary trends (for an excellent review of many of these debates, see Livingstone, 1992, pp. 304-346). First, the prevailing view of the relationship between socioeconomic process and spatial form has changed in urban geography over the last forty years. Through the 1950s, research in the tradition of the Chicago School portrayed the spatial form of the city as a reflection of societal change in the context of dynamic regional and international migration streams and rapid urban industrialization. By the mid-1960s, however, many urbanists had embraced the theories, methods, and foundational assumptions of neoclassical economics, which reduced geography to the status of a passive backdrop on which economic behavior inscribed an efficient spatial organization. Partially in response to this trend, the late 1960s saw a countervailing emphasis on the role of spatial factors in containing, guiding, or constraining economic and social processes. Spatial interaction modeling and time-geographic and behavioral studies all influenced theories of residential differentiation.

During this period, conceptual shifts in the link between social process and spatial form were especially important in the subfield of factorial ecology, in which sophisticated multivariate methods were applied to large secondary datasets in order to derive generalized statements on the residential structure of urban areas. Factorial ecology initially relied on Chicago School theories to explain the underlying processes and was especially indebted to Shevky and Bell's (1955) social area analysis. Shevky and Bell proposed that the spatial imprint of urban industrialization could be understood in terms of three essential constructs: *economic status* variations were rooted in changes in the division of labor; *urbanization* or *family status* variations emerged with new forms of household arrangements; and *segregation* or *ethnic status* variations were attributed to immigration and racial conflicts. The contribution of the empirical studies inspired by these ideas was to show how social divisions took spatial form as transportation innovations allowed the city to expand outward in successive waves of metropolitan growth. Factorial ecology replaced the strict deductive reasoning of social area analysis with a more extensive, inductive search for commonalities in spatial organization among various characteristics of neighborhoods. As a consequence, much of the research in factorial ecology began to focus on technical and empirical questions of spatial pattern -- obscuring the underlying societal dynamics responsible for observed urban geographies (Pratt and Hanson, 1988; Gottdiener, 1985).

Political economy brought a strong reaction against the descriptive, empiricist turn of spatial science (Harvey, 1973). Structuralists sought to replace sophisticated quantitative descriptions of urban structure with analyses rooted in theories of class formation and class conflict. Thus for Harvey (1973, 1985) and others, the particular form of residential differentiation at any historical moment was to be understood as conducive to the continued reproduction of capitalist social relations (see also Badcock, 1994; Harris, 1984; Scott, 1988). Residential geographies emerged from workplace-based relations of production, thereby reproducing class relations (through intergenerational inequalities in education, income, and wealth) and fragmenting class consciousness (as in fights over school busing or affordable housing) (Harvey, 1985). By the mid-1980s, urban geography had engaged theories of the interdependence and mutual constitution of urban society and urban space in a "socio-spatial dialectic" (Soja, 1980; Knox, 1991, 1993). Feminist urban research during the past two decades has further transformed the study of residential differentiation: societal struggles over gender relations are manifest in the built environment, but the resulting urban spaces and places also shape the possibilities for different ways of living (Domosh, 1998; Hayden, 1984; Rose and Villeneuve, 1998).

While the first trend has dealt with the relationship between society and space, the second trend has been methodological. Economic and social geography have witnessed a relative (if not absolute) decline in quantitative and positivist approaches. In studies of urban residential structure, the dominant quantitative methods of the 1960s and 1970s have been largely supplanted by qualitative and case-study approaches. With few exceptions (Davies and Murdie, 1991; Perle, 1981, 1998), there has been little attempt systematically to evaluate changes in patterns of residential differentiation since the high-water mark of ecological research in the late 1960s. Consequently, most of the findings and methods of the factorial ecology literature now seem to be viewed as part of the conventional wisdom of introductory textbooks, or as irrelevant to any understanding of contemporary metropolitan regions in postindustrial western societies. Most such studies were based on data for central cities in 1960 or 1970, thus providing little insight on increasingly variegated suburban neighborhoods during the 1980s (Baldassare, 1994). Most importantly, much of the factorial ecology literature was implicitly based on theories of the behavior of individuals and households in connecting "social space" to "housing space" (Berry and Kasarda, 1977). As such, this literature lent itself to a view of residential differentiation as a natural byproduct of competition in a modern, efficient urban society. This line of reasoning has come under severe and sustained attack (Timms, 1971; Davies, 1984; Berry and Kasarda, 1977; cf.

Harvey, 1973, 1985). By the mid-1980s, the expanding array of conflicting paradigms in urban research seemed to have rendered factorial ecology research doubly irrelevant: it appeared methodologically primitive when viewed from the standpoint of contemporary spatial statistics and geographic information systems (Anselin, 1988; Belsky et al., 1998; Thrall, 1998) and it was seen as substantively and theoretically constrained when viewed from the vantage point of structuralist and poststructuralist approaches. Ironically, the eclipse of the factorial ecology tradition in basic theoretical inquiry coincides with a growing popularity of factorial methods in specialized private-sector and policy-oriented research, much of it enabled by the diffusion of desktop GIS and proprietary databases. A thriving "geodemographic marketing" industry has been built on the use of quantitative-revolution techniques to classify consumer markets according to individual or neighborhood demographic characteristics and spending behavior, and to target these markets with telemarketing or automated direct mail advertising campaigns (Weiss, 1988; Goss, 1992; see also Curry, 1997).[5] Similar approaches are increasingly in vogue in the real estate industry (Birkin and Clark, 1998), in efforts to regulate financial services (Belsky and Clark, 1998; Thrall, 1998), and even to identify "suburban urbanites" who may be lured back to city living (Lang et al., 1997).

These two facets of the reorientation of urban economic and social research -- the problematized link between social and spatial structure and the critique of spatial-analytic methods -- have greatly advanced our understanding of contemporary processes of urban restructuring and social change. For three reasons, however, I believe a quantitative re valuation of residential differentiation is justified. First, the heavy infusion of social theory into postmodern urbanism and other urban literatures has been quick to emphasize contemporary socio-cultural change while obscuring any long-term stability that might be present in urban land use patterns (see Danielson and Wolpert, 1994). As observers of a nascent postmodern urbanism (e.g., Knox, 1991, Dear and Flusty, 1998) readily acknowledge, analysis of new elements of the urban landscape must be balanced with a recognition of the long-run stability of the structural imperatives and individual preferences associated with the mid-century industrial metropolis. Indeed, even as urban theory has sought to escape the spatial determinism of earlier generations, urban policy and urban development seem to be going in precisely the opposite direction. The familiar litany of contradictions of metropolitan growth -- municipal fragmentation, exclusionary zoning, distorted infrastructural funding, endemic fiscal disparities -- remain firmly in place along with the twin pillars of American urban spatial structure: the automobile and the detached, single-family suburban house. Current efforts to disperse low-income populations and to increase the homeownership rate -- now at its highest level ever -- must also be interpreted in this light, given the structured financial incentives of homeownership in suburbia (Stegman et al., 1991; Stegman, 1997).[6] And while postindustrial supply- and demand-side forces may have altered facets of residential development at the high end of the housing market, for example, urban processes associated with the industrial city -- ecological filtering as well as segregation and social reproduction of class relations -- prevail throughout much of the remainder of the urban landscape (Rondinelli et al., 1998; Berry and Kasarda, 1977; Harvey, 1985; Adams, 1988; Pratt and Hanson, 1988).

A quantitative re valuation of residential differentiation is justified for a second reason. Most analysts writing in the postmodern urbanist and global cities literatures draw broad generalizations from arguably distinctive cases of urban restructuring. This contextual sensitivity lends a sort of historical continuity to much of the literature in urban geography. If the legacy of the Chicago School forged American urban geography in the image of the mid-continental industrial city, recent decades have brought a decisive shift to equally distinctive settings for the Los Angeles School and the World City Hypothesis (Scott and Soja, 1996; Sassen, 1994; Friedmann, 1995; Knox and Taylor, 1995).

Finally, the postmodern urbanist and global cities literatures are particularly vulnerable to misinterpretations with regard to the magnitude of seemingly new phenomena. Postmodern urbanists, for example, portray the appearance of new elements of the residential landscape as a reflection of the distinctive 'habitus' of new social classes. While the attention devoted to these groups might suggest that they comprise a large and growing share of the national workforce, empirical evidence is more ambiguous. The occupations Knox (1991) takes as representative of the new bourgeoisie and new *petit* bourgeoisie comprise 15.1% of the national civilian workforce, but this figure drops to 8.1% if school teachers, secretaries, and typists are excluded (U.S. Bureau of the Census, 1997, p. 405). The projected leading occupations in terms of absolute job growth between 1994 and 2005 are cashiers, retail salespersons, janitors, and cleaners (including maids and housekeepers) (U.S. Bureau of the Census, 1997, p. 408). To be sure, the polarization associated with the growth of secondary-sector service workers alongside an expanding class of white-

[5] The intellectual geneology from academic factorial ecology to private-sector marketing is both direct and explicit, summarized in titles such as "The clustering of America" (Weiss, 1988) and "From human ecology to customer targeting: The evolution of geodemographics" (Batey and Brown, 1995).

[6] Federal housing policy has been moving decisively towards devolution, privatization, and market discipline for nearly a decade since the passage of the National Affordable Housing Act of 1990 and the inauguration of the Bush Administration's Homeownership Opportunities for People Everywhere (HOPE) initiatives. In the early 1990s, Housing and Urban Development officials stressed that "Every initiative will be designed to enhance the possibility of homeownership, even the rental assistance programs" (Stegman et al., 1991, p. 162). The Clinton Administration has accelerated these efforts, and is committed to the idea that "homeownership strengthens families and communities, and promotes economic opportunity and personal responsibility" (Stegman, 1995, p. 1604; see also HUD, 1996; Quercia and Galster, 1996).

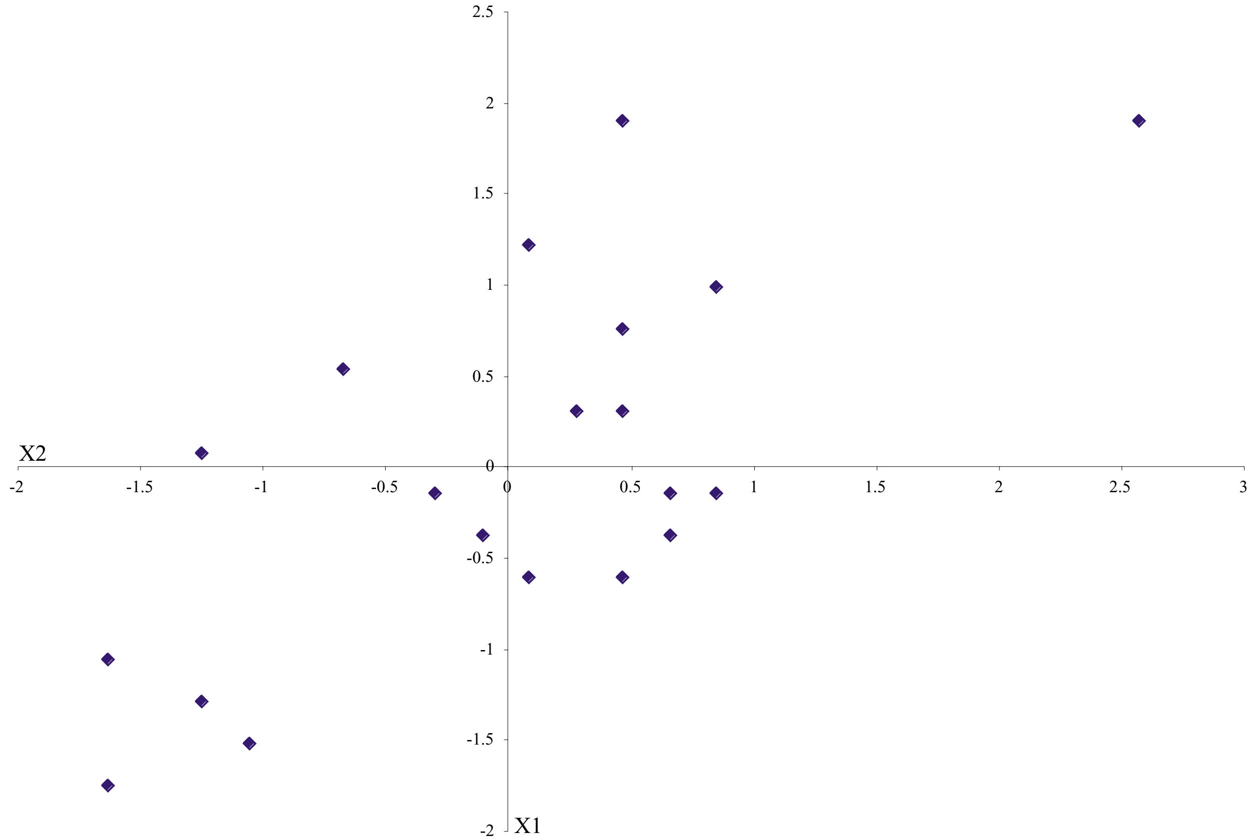
collar elites is a central theme in much of this literature. In a lucid and incisive synthesis of influential globalization theories, however, Storper (1997) highlights numerous weaknesses in the ways globalization has been conceptualized, and in the ways it has been purported to alter the foundations of urban growth and urban inequality. Storper is, for example, highly critical of the "global-dual city" hypothesis advanced by Sassen, Castells, and others: evidence on occupational restructuring and wage rates is "hardly the catastrophic picture painted by the global-dual city theory," and "The 'yuppie plus servant classes' analysis of inequality does not work" (Storper, 1997, p. 232; see also Tickell, 1998). Storper's critique is only partly empirical; more broadly, he argues that occupational restructuring is occurring everywhere, and is not limited to world cities or even large cities. I would add that if there is nothing uniquely urban about occupational polarization, then there is little reason to believe that globalization-driven labor market changes have produced completely new residential structures. The evidence marshalled so far seems insufficient to justify the conclusion that the American city has "lost its neat social patches of the 1950s" (Kirby, 1989, cited in Knox, 1991) to be replaced by a restless, 'glocalized' postindustrial landscape.

3. A Simple Geometric View of Principal Components Analysis.

Consider a simple dataset with twenty observations, and two variables. It is a simple matter to express each observation in terms of its distance in standard deviations from the respective variable mean. By definition, these Z-scores have a mean of 0 and a variance of 1, so the total variance for our two-variable, standardized dataset is 2.00.

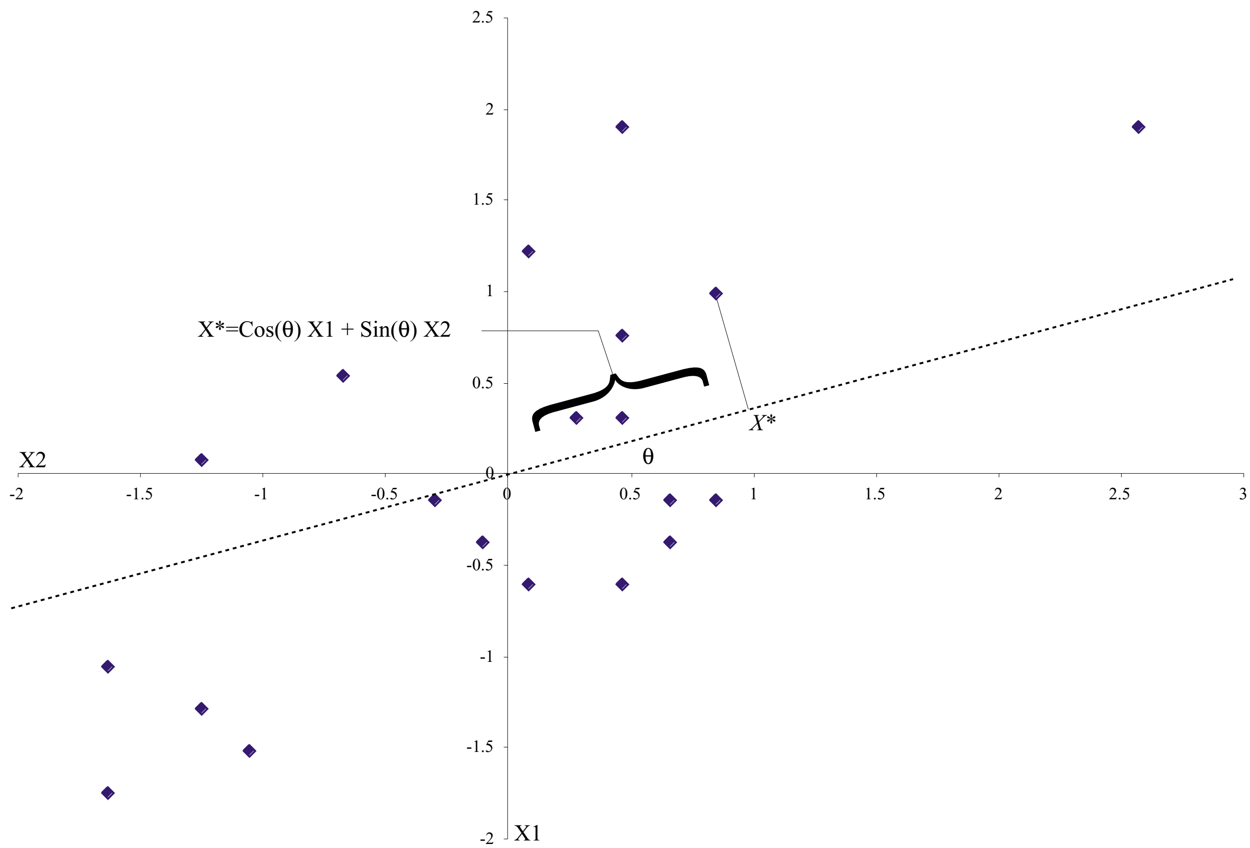
Observation	X1	Mean-Corrected	Z-score	X2	Mean-Corrected	Z-score
1	16	8.35	1.90	8	2.45	0.47
2	12	4.35	0.99	10	4.45	0.85
3	13	5.35	1.22	6	0.45	0.09
4	10	2.35	0.54	2	-3.55	-0.68
5	9	1.35	0.31	8	2.45	0.47
6	8	0.35	0.08	-1	-6.55	-1.25
7	7	-0.65	-0.15	4	-1.55	-0.30
8	5	-2.65	-0.60	6	0.45	0.09
9	3	-4.65	-1.06	-3	-8.55	-1.63
10	2	-5.65	-1.29	-1	-6.55	-1.25
11	0	-7.65	-1.74	-3	-8.55	-1.63
12	1	-6.65	-1.52	0	-5.55	-1.06
13	11	3.35	0.76	8	2.45	0.47
14	16	8.35	1.90	19	13.45	2.57
15	9	1.35	0.31	7	1.45	0.28
16	6	-1.65	-0.38	5	-0.55	-0.10
17	7	-0.65	-0.15	9	3.45	0.66
18	7	-0.65	-0.15	10	4.45	0.85
19	6	-1.65	-0.38	9	3.45	0.66
20	5	-2.65	-0.60	8	2.45	0.47
Mean	7.65			5.55		
Variance	19.23		1.00	27.45		1.00
Std.deviation	4.39			5.24		

Now consider the graph of these two variables (see the next page). A simple glance at the scatter of points suggests a fairly strong, although far from perfect, correlation. If we were to draw a line passing through this scatter of points, could we obtain another axis to describe the information in this dataset?



The principles of geometry come in handy at this point. If we lay in another axis through the origin at any angle, represented as θ, the mathematical formulas for the sine, cosine, and tangent of a right-angle triange can help us to work out several key relationships. Specifically, the perpendicular projection of a point on the new axis intersects at a point whose distance from the origin can be expressed as $X^* = \text{Cos}(\theta)\, X1 + \text{Sin}(\theta)\, X2$.[7] (See the graph on the following page).

[7] For the derivation of this and similar equations, see an intermediate mathematics or geometry text. There is also a short summary in Chapter 2 of Sharma, Subhash (1996). *Applied Multivariate Techniques.* New York: John Wiley and Sons.



For any given value of theta, then, it is a simple matter to work out the values of X* for each of our twenty observations. When θ is 5 degrees, for example, the calculations are:

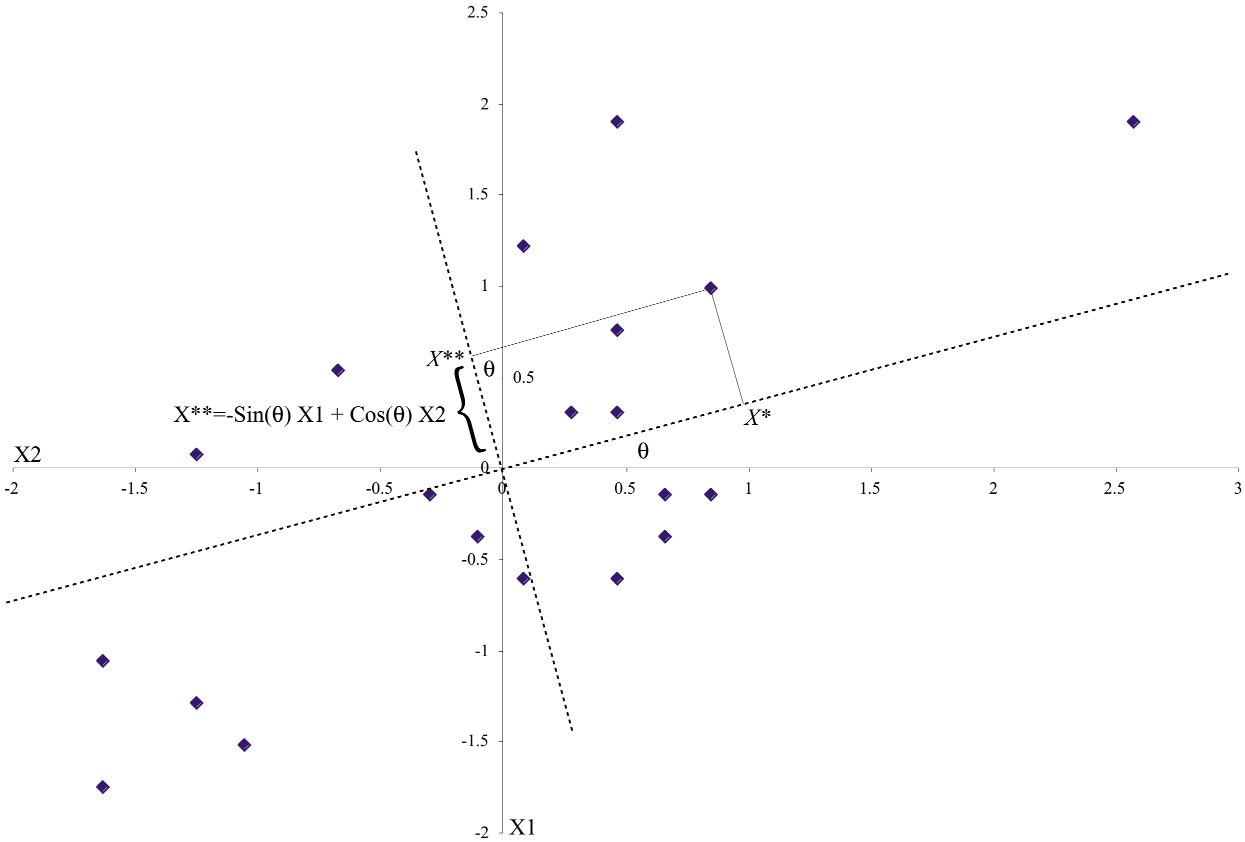
X1	X2	New Variable X*
1.90	0.47	1.94
0.99	0.85	1.06
1.22	0.09	1.22
0.54	-0.68	0.47
0.31	0.47	0.35
0.08	-1.25	-0.03
-0.15	-0.30	-0.17
-0.60	0.09	-0.59
-1.06	-1.63	-1.20
-1.29	-1.25	-1.39
-1.74	-1.63	-1.88
-1.52	-1.06	-1.60
0.76	0.47	0.80
1.90	2.57	2.12
0.31	0.28	0.33
-0.38	-0.10	-0.38
-0.15	0.66	-0.09
-0.15	0.85	-0.07
-0.38	0.66	-0.32
-0.60	0.47	-0.56
Variance		1.12

Note that each of the original variables has a variance of 1.0, but the variance of the new axis is 1.12, which constitutes more than half of the total variance for the entire dataset (e.g., 1.12/2.00 or 56%). Each value of theta will yield a different set of scores on X*, and will also result in distinct values for the variance term. If we calculate transformed values and variances for different values of theta, we can compare the variance of the new axis to the total for our dataset. Note that as we increase the angle, the new variable accounts for an increasing fraction of total variance, until 45 degrees, and then declines; by the time theta is 90 degrees, the new axis is equivalent to X2, and, not surprisingly, its proportion of variance is back to 1.00 or 50.0%.

Theta	Variance of X*	Proportion
5	1.121	56.0%
10	1.238	61.9%
15	1.348	67.4%
20	1.447	72.4%
25	1.533	76.7%
30	1.603	80.1%
35	1.654	82.7%
40	1.685	84.3%
45	1.696	84.8%
50	1.685	84.3%
55	1.654	82.7%
60	1.603	80.1%
65	1.533	76.7%
70	1.447	72.4%
75	1.348	67.4%
80	1.238	61.9%
85	1.121	56.0%
90	1.000	50.0%

Thus far, we have found that X* accounts for the largest proportion of variance when the axis is at an angle of 45 degrees. Still, this new axis does not capture all of the variance in the original dataset. We can lay in another axis, orthogonal to X*, to try to capture more of this variation. If we call this second new variable X**, the projection of observations on this axis will be defined by $X^{**}=-\sin(\theta)X1 + \cos(\theta)X2$. So when theta is 45 degrees, the coordinates on X** are:

X1	X2	X**
1.90	0.47	-1.02
0.99	0.85	-0.10
1.22	0.09	-0.80
0.54	-0.68	-0.86
0.31	0.47	0.11
0.08	-1.25	-0.94
-0.15	-0.30	-0.10
-0.60	0.09	0.49
-1.06	-1.63	-0.40
-1.29	-1.25	0.03
-1.74	-1.63	0.08
-1.52	-1.06	0.32
0.76	0.47	-0.21
1.90	2.57	0.47
0.31	0.28	-0.02
-0.38	-0.10	0.19
-0.15	0.66	0.57
-0.15	0.85	0.71
-0.38	0.66	0.73
-0.60	0.47	0.76
Variance		0.3038



Note that this time, the variance term is much smaller than that of the each of the original variables. But the variances of the two new axes sum to 2.0 -- the total variation in the original dataset. This is the basic approach of principal components analysis: obtaining linear combinations of variables into new axes, such that the first one accounts for the largest share of total variance, the second is orthogonal to the first and accounts for less variance, etc. Several properties hold for these components:

1. The maximum number of new components is equal to the number of original variables.

2. The cumulative variance of all components sums to the same value as the total variance in the original dataset.

3. Components are orthogonal to one another, and are therefore uncorrelated.

4. The correlation between the original variable X1 and any of the components is called a ***loading***. The sum of the squared loadings for each component is referred to as the ***communality*** of the variable with the components.

5. The coordinates of observations on the new axes are known as the ***component scores***. These are the values shown in the X** column above.

4. An Algebraic Solution.

Although geometric principles help us to visualize principal components, this can be a bit cumbersome. Moreover, it won't get us beyond three dimensions. Matrix algebra does not have these limitations, and turns out to be equivalent to the geometric solution.

Let's return to the original two variables, X1 and X2. Rather than graphing them, let's compute their correlation coefficient:

$$r = \frac{\sum (Y - \mu_Y)(X - \mu_X)}{\sqrt{\sum (Y - \mu_Y)^2 \sum (X - \mu_X)^2}}$$

For our dataset, r=0.696, or, to keep a level of precision that will be helpful in subsequent calculations, 0.69615116. So X1 and X2 can be represented as a square, two-by-two correlation matrix, which we will call A:

$$A = \begin{pmatrix} 1.0 & .69615116 \\ .69615116 & 1.0 \end{pmatrix}$$

The field of matrix algebra gives us a number of valuable principles when data are arranged in this form. One of these principles is the ***inverse***: if A is invertible, it means there is another square matrix (which we will call A^{-1}) which, when multiplied by A, gives us the identity matrix:

A * A-1 = I

Where I, the identity matrix, is simply

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Recall that, in matrix algebra, the identity matrix performs many of the same functions as the value of 1.

Not all matrices are invertible. In fact, a matrix has an inverse if and only if the differences between the cross-products is non-zero. That is, for the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

an inverse exists if and only if (a*d)-(b*c)›0. This value -- the cross-product of a square matrix -- is called the ***determinant***. For our dataset, the calculations give us (1*1)-(0.696151156*0.696151156), or 1-0.48462644, or 0.51537356. Since this is a non-zero value, we know that our matrix is invertible. To keep things clear, when referring to the determinant we replace the curved brackets with straight ones, so det(A) is the same as

$$\det\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

The matrix inverse and determinant are related by the principle of the *characteristic root* of a matrix. Every square matrix has a unique polynomial equation that describes it. This characteristic equation is of the same order as the matrix, so that a 2 x 2 matrix has a quadratic equation, a 3 x 3 has a cubic, and so forth. The equation is obtained from the following property of invertible square matrices:

$\det(A - \lambda I) = 0$

Where A is our original matrix, λ is a scalar (an n-by-n matrix with a constant value, λ, in every cell), and I is the identity matrix. Another way of expressing the same thing is to take the determinant of a matrix X with λ subtracted from the diagonal elements.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
$$\det\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

So now consider the correlation matrix describing the correlation of our two variables.

$$A = \begin{pmatrix} 1.0 & 0.69615116 \\ 0.69615116 & 1.0 \end{pmatrix}$$
$$\det\begin{vmatrix} 1-\lambda & 0.69615116 \\ 0.69615116 & 1-\lambda \end{vmatrix} = 0$$

Recall that the determinant is nothing more than the cross-product. So our equation is

$(1-\lambda)(1-\lambda) - (0.48462644) = 0$

When we multiply the first two terms, we get

$1 + \lambda^2 - \lambda - \lambda$, so the equation simplifies to

$(\lambda^2 - 2\lambda + 1) - (0.4846244) = 0$

$\lambda^2 - 2\lambda + 0.51537356 = 0$

This equation is now in a form that provides incontrovertible proof that all those hours you spent in algebra and trigonometry, wherever you first encountered these lovely subjects in your academic career, were not wasted. That is because the binomial theorem gives us

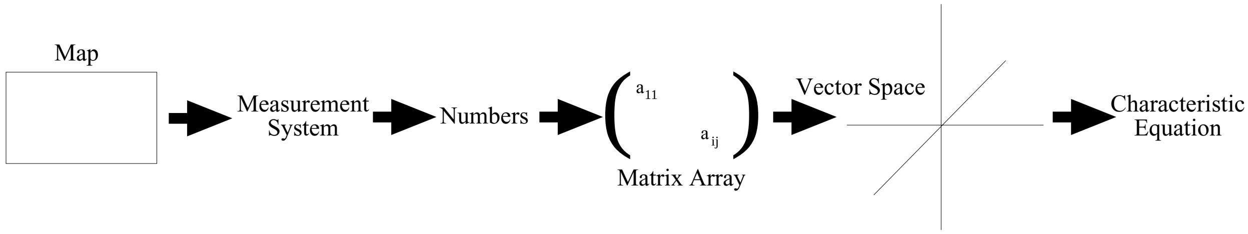
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where a=1, b=2, and c=0.51537356. It's been a while, hasn't it? I had to look in my old notes to find it, too. So the equation can be solved like this:

$$\lambda = \frac{2 \pm \sqrt{2^2 - 4(0.51537356)}}{2}$$
$$\lambda = \frac{2 \pm \sqrt{4 - 2.06149424}}{2}$$
$$\lambda = \frac{2 \pm \sqrt{1.93850576}}{2}$$
$$\lambda = \frac{2 \pm 1.39230232}{2}$$
$$\lambda_1 = 1.69615116, \lambda_2 = 0.30384884$$

These values should look familiar: they are the variance terms calculated for the two principal components we extracted in the geometric example shown above. These are called ***eigenvalues***, *latent roots*, or *characteristic roots* of the correlation matrix; *eigen* is German for "root."
Peter Gould suggested that many geographical problems could be conceptualized with these simple techniques.[8] Writing at a time when the use of quantitative techniques in the field was a haphazard blend of inductive and deductive approaches, Gould was part of a group of scholars advocating the use of quantification to find order amidst complex spatial patterns. When confronted with a particularly complex map, the geographer could use principles drawn from mathematics, geometry, algebra, and statistics to uncover latent structure. For example:

[8] See Gould, Peter (1967). "On the Geographical Interpretation of Eigenvalues." *Transactions of the Institute of British Geographers*, Winter, 53-86.



It is worth examining this analytical sequence in light of contemporary trends in the physical and social sciences. First, there has been a growing recognition of complexity, contingency, and chaos in many natural and social systems. Chaos and complexity do not imply that understanding or generalization are impossible; but there is a much greater suspicion of the endeavor to find a simple "latent structure." Second, Gould's recommendation is, in many respects, a complete reversal of how many geographers now approach cartographic problems. Particularly in GIScience applications, the map is often the goal of the entire analysis, and marks the *end* of the procedure rather than its beginning.

5. Factor Analysis.

Principal components analysis is often confused with factor analysis. Although the procedures are similar, the analytical frameworks are based on a fundamentally distinct set of assumptions. PCA is nothing more than a linear recombination of original variables into a new set of variables, each of which is orthogonal to one another. All of the variance in the original dataset is simply re-allocated among the new measures. Factor analysis, by contrast, is premised on the distinction between common and unique variance. As noted earlier, the technique was developed in the field of educational psychology, where analysts sought to disentangle two underlying aspects of differences among student achievement: a "general" construct of intelligence or ability, and a specific aptitude that was unique to a particular subject area. So if we had a sample of student scores on a math examination, the concern would be to distinguish a unique, quantitative aptitude from the general underlying construct of intelligence.

Some analysts fail to clarify this distinction, and treat factor analysis the same as PCA. PCA can be understood as a special case of factor analysis -- where all of the variance is common, and can be reallocated to the new dimensions. Principal components are usually, but not always, the first step in a factor analysis. Once the Eigenvalues are calculated for the components, the analyst typically chooses how many components are required to summarize the information in the original dataset: for many applications, three or four components will suffice to capture the variance in the original dataset.

These selected components are then subjected to another transformation, or "rotation," that is designed to facilitate interpretation. In conceptual terms, selecting a subset of all n components for rotation has the effect of reducing our n-dimensional space by the number of components discarded. The most common rotation method is "varimax," which keeps the components orthogonal to one another, and is designed to make each of the original variables load on one, and only one of the new factors. There are scores of other rotations, and none is "objectively" correct or incorrect. One of the more complex possibilities is the use of oblique factor rotations, which allow the components to move away from the orthogonal, right-angle restriction. Interpreting oblique factor solutions is much more complicated than orthogonal analyses, however.

6. PCA and Factor Analysis in SAS.

SAS includes extensive facilities for principal components analysis, but the basic analysis is remarkably simple:

PCA.SAS

```
***
libname qga "c:\sasproj\qga";
data qga.pca;
        input x1 x2;
        cards;
16 8
12 10
13 6
10 2
9 8
8 -1
7 4
5 6
3 -3
2 -1
0 -3
1 0
11 8
16 19
9 7
6 5
7 9
7 10
6 9
5 8
;
proc princomp out=new;
var x1 x2;
run;

proc print;
var x1 x2 prin1 prin2;
run;

***
```

This code gives us the following output:

PCA Output

```
The SAS System         15:47 Monday, March 19, 2001   7

                                Principal Component Analysis
```

```
20 Observations
 2 Variables

                                  Simple Statistics

                                          X1                  X2

                      Mean       7.650000000       5.550000000
                      StD        4.498830257       5.375137697

                                 Correlation Matrix

                                         X1          X2

                             X1      1.0000      0.6962
                             X2      0.6962      1.0000

                        Eigenvalues of the Correlation Matrix

                   Eigenvalue      Difference      Proportion      Cumulative

        PRIN1         1.69615         1.39230        0.848076         0.84808
        PRIN2         0.30385          .             0.151924         1.00000

                                    Eigenvectors

                                      PRIN1         PRIN2

                           X1      0.707107      0.707107
                           X2      0.707107      -.707107
                                   The SAS System         15:47 Monday, March 19, 2001   8

                      OBS    X1     X2      PRIN1       PRIN2

                        1    16      8     1.63472     0.99012
                        2    12     10     1.26912     0.09831
                        3    13      6     0.90009     0.78169
                        4    10      2    -0.09764     0.83637
                        5     9      8     0.53449    -0.11011
                        6     8     -1    -0.80665     0.91667
                        7     7      4    -0.30607     0.10174
                        8     5      6    -0.35732    -0.47571
                        9     3     -3    -1.85563     0.39390
                       10     2     -1    -1.74970    -0.02638
                       11     0     -3    -2.32716    -0.07763
                       12     1      0    -1.77533    -0.31511
                       13    11      8     0.84884     0.20424
                       14    16     19     3.08178    -0.45695
                       15     9      7     0.40294     0.02144
                       16     6      5    -0.33169    -0.18699
                       17     7      9     0.35169    -0.55602
                       18     7     10     0.48324    -0.68757
                       19     6      9     0.19451    -0.71319
                       20     5      8    -0.09421    -0.73882
```

Now we could, of course, spend a bit of time discussing the characteristics of this nice little example, derived from that little two-by-two matrix for which we have the worked example above. Still, aren't we getting just a little bit bored with the hypothetical dataset? Stretch your imagination from the sterile, two-dimensional graph shown above, and consider the thirty-five dimensional world I imposed on the Twin Cities of Minneapolis and St. Paul. The following is the code and the output I used for the first batch of models in the "Continuity and Change" article. Read the article first to understand what the long list of variables is actually trying to measure.

Factor.SAS

```
libname tcfact "c:\sasproj\tcfact";
```

```
data tcfact.subset;

        set tcfact.tc8090_2;
        run;
proc factor data=tcfact.subset priors=smc n=8 scree rotate=varimax;
title 'principal factor analysis, 1980';
var
FINCRAT8
WHTPR8

KIDS8
OLD8
CHILD8
BOOM8
MAGE18
MAGE28

RGRWTH18
RGRWTH28
RGRWTH38
RGRWTH48
HSG708
HSG608
HSG508

FNVMARX8
FMRSPPX8
FDIVORX8
WFHNK8
WFHWK8
MNVMARX8
MMRSPPX8
MDIVORX8
WMHNK8
WMHWK8
WMCNK8
WMCWK8

WHSG8
WCOL8
WCGR8

WFEMPX8
WFUEMPX8
FNOPART8

WMEMPX8
WMUEMPX8
MNOPART8

PROF8
MGRS8
SALE8
ADMN8
CRFT8
OPER8
SERV8;
run;

proc factor data=tcfact.subset priors=smc n=8 scree rotate=varimax;
title 'principal factor analysis, 1990';
var
FINCRAT9
WHTPR9

KIDS9
OLD9
CHILD9
BOOM9
MAGE19
MAGE29

RGRWTH19
RGRWTH29
RGRWTH39
RGRWTH49
HSG709
HSG609
HSG509

FNVMARX9
FMRSPPX9
```

```
FDIVORX9
WFHNK9
WFHWK9
MNVMARX9
MMRSPPX9
MDIVORX9
WMHNK9
WMHWK9
WMCNK9
WMCWK9

WHSG9
WCOL9
WCGR9

WFEMPX9
WFUEMPX9
FNOPART9

WMEMPX9
WMUEMPX9
MNOPART9

PROF9
MGRS9
SALE9
ADMN9
CRFT9
OPER9
SERV9;
run;
```

Output of Factor.SAS

```
principal factor analysis, 1980                               1
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Initial Factor Method: Principal Factors

                              Prior Communality Estimates: SMC

     FINCRAT8    WHTPR8     KIDS8      OLD8     CHILD8      BOOM8     MAGE18     MAGE28   RGRWTH18
     0.841581  0.829070  0.880302  0.953485  0.968040  0.944748  0.834833  0.820607  0.901506

     RGRWTH28  RGRWTH38  RGRWTH48    HSG708    HSG608     HSG508   FNVMARX8   FMRSPPX8   FDIVORX8
     0.786292  0.918517  0.857907  0.807807  0.475517  0.628200  0.926802  0.971306  0.797071

       WFHNK8    WFHWK8  MNVMARX8  MMRSPPX8  MDIVORX8     WMHNK8     WMHWK8     WMCNK8     WMCWK8
     0.474728  0.647830  0.974908  0.986530  0.896730  0.297188  0.282847  0.726821  0.775141

        WHSG8     WCOL8     WCGR8   WFEMPX8  WFUEMPX8   FNOPART8    WMEMPX8   WMUEMPX8   MNOPART8
     0.925625  0.813981  0.916245  0.954767  0.371552  0.837233  0.967599  0.498280  0.897611

        PROF8     MGRS8     SALE8     ADMN8     CRFT8      OPER8      SERV8
     0.854318  0.843235  0.760683  0.709590  0.788424  0.831144  0.495285

    Eigenvalues of the Reduced Correlation Matrix:  Total = 33.6718876  Average = 0.78306715

                      1            2            3            4            5            6            7
Eigenvalue      10.2141       5.7645       4.6851       3.2723       1.7810       1.3459       1.1935
Difference       4.4496       1.0793       1.4128       1.4913       0.4352       0.1524       0.0758
Proportion       0.3033       0.1712       0.1391       0.0972       0.0529       0.0400       0.0354
Cumulative       0.3033       0.4745       0.6137       0.7109       0.7638       0.8037       0.8392

                      8            9           10           11           12           13           14
Eigenvalue       1.1177       0.9140       0.7641       0.7045       0.4487       0.3946       0.3284
Difference       0.2037       0.1499       0.0596       0.2558       0.0541       0.0662       0.0226
Proportion       0.0332       0.0271       0.0227       0.0209       0.0133       0.0117       0.0098
Cumulative       0.8724       0.8995       0.9222       0.9431       0.9564       0.9682       0.9779

                     15           16           17           18           19           20           21
Eigenvalue       0.3058       0.2728       0.2208       0.1862       0.1608       0.1255       0.1111
Difference       0.0329       0.0520       0.0346       0.0254       0.0354       0.0143       0.0192
Proportion       0.0091       0.0081       0.0066       0.0055       0.0048       0.0037       0.0033
Cumulative       0.9870       0.9951       1.0017       1.0072       1.0120       1.0157       1.0190

                     22           23           24           25           26           27           28
Eigenvalue       0.0919       0.0818       0.0606       0.0400       0.0298       0.0197       0.0181
Difference       0.0101       0.0212       0.0206       0.0102       0.0100       0.0016       0.0304
Proportion       0.0027       0.0024       0.0018       0.0012       0.0009       0.0006       0.0005
Cumulative       1.0217       1.0242       1.0260       1.0271       1.0280       1.0286       1.0292
```

	29	30	31	32	33	34	35
Eigenvalue	-0.0122	-0.0183	-0.0235	-0.0299	-0.0315	-0.0415	-0.0506
Difference	0.0061	0.0052	0.0064	0.0016	0.0100	0.0091	0.0068
Proportion	-0.0004	-0.0005	-0.0007	-0.0009	-0.0009	-0.0012	-0.0015
Cumulative	1.0288	1.0282	1.0275	1.0267	1.0257	1.0245	1.0230

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Initial Factor Method: Principal Factors

	36	37	38	39	40	41	42
Eigenvalue	-0.0574	-0.0677	-0.0802	-0.0879	-0.0925	-0.1107	-0.1244
Difference	0.0103	0.0124	0.0077	0.0047	0.0182	0.0137	0.0286
Proportion	-0.0017	-0.0020	-0.0024	-0.0026	-0.0027	-0.0033	-0.0037
Cumulative	1.0213	1.0193	1.0169	1.0143	1.0115	1.0082	1.0045

	43
Eigenvalue	-0.1530
Difference	
Proportion	-0.0045
Cumulative	1.0000

8 factors will be retained by the NFACTOR criterion.

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Initial Factor Method: Principal Factors

Scree Plot of Eigenvalues

```
     Ç
  12 à
     Ç
     Ç
     Ç
     Ç
     Ç      1
  10 à
     Ç
     Ç
     Ç
     Ç
     Ç
   8 à
     Ç
     Ç
E    Ç
i    Ç
g    Ç
e  6 à
n    Ç        2
v    Ç
a    Ç
l    Ç         3
u    Ç
e  4 à
s    Ç
     Ç           4
     Ç
     Ç
     Ç
   2 à
     Ç             5
     Ç               6 7
     Ç                  8 9
     Ç                      0 1
     Ç                          23 4 5 6 78
   0 à                                      9 0 1 23 4 5 6 78 9 0 1 23 4 5 6 78 9 0 1 23
     Ç
     Ç
     Ç
     Ç
     Ç
  -2 à
     äÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉÉÉÉàÉÉÉÉÉ
          0        5       10       15       20       25       30       35       40       45

                                                 Number
```

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Initial Factor Method: Principal Factors

Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	
FINCRAT8	-0.67571	-0.55007	0.01516	-0.07511	
WHTPR8	0.77918	0.10095	0.02098	-0.08301	pct of white pers in fam unrelind belw p
KIDS8	-0.19629	0.65730	0.34882	-0.35483	percentage of total pop under 6 years ol
OLD8	0.48817	-0.26977	-0.50267	0.07643	percentage of total pop over 64 years ol
CHILD8	-0.63333	0.53204	0.23683	-0.27861	percentage of total pop under 18 years o
BOOM8	0.63196	-0.03847	0.54443	-0.06738	percentage of persons born 1945-1960
MAGE18	-0.43433	-0.25151	-0.36152	0.36276	percentage of persons aged 45-54
MAGE28	0.01056	-0.24760	-0.63547	0.37630	percentage of persons aged 55-64
RGRWTH18	0.64314	-0.01839	0.46940	-0.15844	pct of occ hsg units head moved 1979-198
RGRWTH28	-0.07528	0.06379	0.46546	-0.57194	pct of occ hsg units head moved 1975-197
RGRWTH38	0.77880	-0.08172	0.29218	-0.02068	pct of ohu rental head moved 1979-1980
RGRWTH48	0.80826	-0.11478	0.07958	-0.09117	pct of ohu rental head moved 1975-1978
HSG708	-0.34140	0.09889	0.47091	-0.54064	pct of occ hsg units built in 70s
HSG608	-0.18675	0.00455	0.21773	0.03430	pct of occ hsg units built in 60s
HSG508	-0.29828	-0.03607	-0.33165	0.45613	pct of occ hsg units built in 50s
FNVMARX8	0.72452	-0.32148	0.24730	0.06763	pct of females 15 and over never married
FMRSPPX8	-0.92306	0.19504	0.03015	-0.08385	pct of fem15+ married spouse present
FDIVORX8	0.66816	0.11979	0.08249	0.05880	pct of females 15 and over divorced
WFHNK8	0.20545	0.06230	-0.14387	0.35247	pct of white hh f headed fam wo ch 18
WFHWK8	0.22866	0.43144	0.20268	0.15417	pct of white hh f headed fam w ch 18
MNVMARX8	0.79023	-0.21190	0.13970	0.06019	pct of males 15+ never married
MMRSPPX8	-0.92497	0.13414	-0.06007	-0.03791	pct of m 15+ married spouse present
MDIVORX8	0.71806	0.06963	0.03463	0.00687	pct of males 15 and over divorced
WMHNK8	0.18604	0.09210	0.03846	0.18334	pct of white hh m headed fam wo ch 18
WMHWK8	-0.00419	0.22010	0.21520	0.06789	pct of white hh m headed fam w ch 18
WMCNK8	-0.30848	-0.08073	-0.06388	0.44980	pct of white hh mar couple fam wo ch 18
WMCWK8	-0.59903	0.24422	0.30200	0.05092	pct of white hh mar couple fam w ch 18
WHSG8	-0.22263	0.54368	0.25364	0.58611	Pct of whites 25+ high school graduates
WCOL8	-0.11029	-0.35441	0.63759	0.33748	Pct of whites 25+ some coll no degree
WCGR8	-0.06815	-0.73944	0.48203	0.08822	Pct of whites 25+ coll grad and more
WFEMPX8	-0.03539	0.02965	0.70949	0.62933	percentage of white fem 16+ empl
WFUEMPX8	0.09965	0.31430	0.27345	0.11364	percentage of white fem 16+ unempl
FNOPART8	0.18548	0.01939	-0.46000	-0.24826	Pct of females aged 16+ worked <=26 week
WMEMPX8	-0.26404	0.01727	0.63810	0.59130	percentage of white males 16+ empl
WMUEMPX8	0.34611	0.38974	0.18864	0.21429	percentage of white males 16+ unempl
MNOPART8	0.74872	-0.01572	-0.36131	-0.07618	Pct of males aged 16+ worked <=26 weeks
PROF8	-0.01879	-0.72609	0.23998	-0.14022	Professional tech workers as pct of tot
MGRS8	-0.44489	-0.66157	0.09986	-0.13902	Managers administrators as pct of total
SALE8	-0.50158	-0.57894	0.10553	0.02216	Sales workers as pct of total
ADMN8	0.34311	0.14829	-0.01231	0.33575	Admin support wkrs as pct of total
CRFT8	-0.25835	0.70067	-0.12387	0.03944	Craft workers as pct of total
OPER8	0.17099	0.79868	-0.15703	0.01177	Operatives as pct of total
SERV8	0.19246	0.54271	-0.17913	0.08850	Service workers as pct of total

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Initial Factor Method: Principal Factors

Factor Pattern

	FACTOR5	FACTOR6	FACTOR7	FACTOR8	
FINCRAT8	0.05464	0.20872	0.12056	-0.02273	
WHTPR8	0.21462	0.07478	0.23363	-0.10264	pct of white pers in fam unrelind belw p
KIDS8	0.12490	-0.04842	-0.09288	0.03660	percentage of total pop under 6 years ol
OLD8	0.39826	-0.29423	-0.00614	0.29441	percentage of total pop over 64 years ol
CHILD8	0.09552	0.11756	0.03387	-0.22235	percentage of total pop under 18 years o
BOOM8	-0.27009	-0.13348	-0.25591	0.01669	percentage of persons born 1945-1960
MAGE18	-0.19262	0.40422	0.25108	-0.02220	percentage of persons aged 45-54
MAGE28	-0.02644	0.07279	0.10761	0.23893	percentage of persons aged 55-64
RGRWTH18	-0.14683	0.17574	0.00512	0.20167	pct of occ hsg units head moved 1979-198
RGRWTH28	0.11765	0.00089	0.09713	0.21988	pct of occ hsg units head moved 1975-197
RGRWTH38	-0.15144	0.14937	-0.02064	0.09532	pct of ohu rental head moved 1979-1980
RGRWTH48	0.00162	0.08020	0.07323	0.10000	pct of ohu rental head moved 1975-1978
HSG708	0.04752	0.01353	0.07747	0.26995	pct of occ hsg units built in 70s
HSG608	-0.19334	0.32611	0.19173	-0.00195	pct of occ hsg units built in 60s
HSG508	-0.31540	0.11454	-0.02089	-0.08902	pct of occ hsg units built in 50s
FNVMARX8	-0.19023	0.10203	0.00110	-0.30831	pct of females 15 and over never married
FMRSPPX8	-0.08031	-0.05785	0.02064	0.00970	pct of fem15+ married spouse present
FDIVORX8	0.01847	0.24884	-0.03974	0.27850	pct of females 15 and over divorced
WFHNK8	0.25402	-0.02240	-0.29217	0.01352	pct of white hh f headed fam wo ch 18
WFHWK8	0.22567	0.42689	-0.25162	0.00965	pct of white hh f headed fam w ch 18
MNVMARX8	-0.12874	0.01434	0.04009	-0.38813	pct of males 15+ never married
MMRSPPX8	0.01957	-0.10714	-0.11918	0.14474	pct of m 15+ married spouse present
MDIVORX8	0.05696	0.24670	0.19393	0.30424	pct of males 15 and over divorced
WMHNK8	0.25094	0.11857	-0.23477	-0.09029	pct of white hh m headed fam wo ch 18
WMHWK8	0.17229	0.26261	-0.21682	-0.00047	pct of white hh m headed fam w ch 18

WMCNK8	0.39166	0.26384	-0.35027	0.07285	pct of white hh mar couple fam wo ch 18
WMCWK8	0.28343	0.21691	-0.16950	-0.08604	pct of white hh mar couple fam w ch 18
WHSG8	0.08620	-0.18987	0.22982	0.08042	Pct of whites 25+ high school graduates
WCOL8	0.16146	-0.14368	0.14567	0.11329	Pct of whites 25+ some coll no degree
WCGR8	0.21227	-0.01702	0.04055	-0.17081	Pct of whites 25+ coll grad and more
WFEMPX8	0.01855	-0.17056	0.11372	0.04471	percentage of white fem 16+ empl
WFUEMPX8	0.12228	0.04415	0.16977	-0.14883	percentage of white fem 16+ unempl
FNOPART8	0.56494	-0.07289	0.24943	-0.11083	Pct of females aged 16+ worked <=26 week
WMEMPX8	0.19056	-0.16876	0.25643	0.02628	percentage of white males 16+ empl
WMUEMPX8	0.12745	0.04469	0.15789	-0.08126	percentage of white males 16+ unempl
MNOPART8	0.33148	-0.01629	0.13259	-0.05950	Pct of males aged 16+ worked <=26 weeks
PROF8	0.16903	-0.17132	-0.21313	-0.19825	Professional tech workers as pct of tot
MGRS8	0.12934	0.24207	0.14003	0.15518	Managers administrators as pct of total
SALE8	-0.04939	0.17643	0.05266	0.12964	Sales workers as pct of total
ADMN8	-0.28008	-0.20504	-0.25927	0.28235	Admin support wkrs as pct of total
CRFT8	-0.18206	-0.02904	0.10736	-0.01399	Craft workers as pct of total
OPER8	-0.00972	0.02301	0.15017	-0.03162	Operatives as pct of total
SERV8	0.02347	0.10463	0.03602	-0.08689	Service workers as pct of total

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Initial Factor Method: Principal Factors

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
10.214075	5.764491	4.685143	3.272320	1.781030	1.345852	1.193458	1.117688

Final Communality Estimates: Total = 29.374057

FINCRAT8	WHTPR8	KIDS8	OLD8	CHILD8	BOOM8	MAGE18	MAGE28	RGRWTH18
0.826635	0.741407	0.746065	0.901511	0.891415	0.858334	0.778216	0.681509	0.752542

RGRWTH28	RGRWTH38	RGRWTH48	HSG708	HSG608	HSG508	FNVMARX8	FMRSPPX8	FDIVORX8
0.625135	0.753755	0.702895	0.721691	0.263974	0.529273	0.835664	0.908333	0.612446

WFHNK8	WFHWK8	MNVMARX8	MMRSPPX8	MDIVORX8	WMHNK8	WMHWK8	WMCNK8	WMCWK8
0.341600	0.599843	0.861533	0.925622	0.715987	0.218486	0.245042	0.659079	0.675789

WHSG8	WCOL8	WCGR8	WFEMPX8	WFUEMPX8	FNOPART8	WMEMPX8	WMUEMPX8	MNOPART8
0.855787	0.738951	0.867718	0.945936	0.264280	0.706980	0.958060	0.402973	0.828451

PROF8	MGRS8	SALE8	ADMN8	CRFT8	OPER8	SERV8
0.747462	0.783910	0.651518	0.520027	0.620284	0.716100	0.391839

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Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2	3	4	5	6	7	8
1	0.93922	-0.17527	-0.03158	-0.07583	0.27611	0.03110	-0.00807	-0.05591
2	-0.16027	-0.92943	0.24882	0.07844	-0.02194	0.20097	-0.03542	-0.01699
3	0.17625	0.23257	0.59341	0.59547	-0.28306	0.15599	-0.26417	-0.18408
4	-0.00880	-0.07235	-0.70601	0.61957	0.04844	0.26693	-0.18978	0.05249
5	-0.08155	0.15096	0.19915	0.18754	0.44164	0.49075	0.66165	0.14857
6	0.18545	0.03252	-0.12764	-0.28098	-0.71437	0.53515	0.16568	0.21248
7	0.13856	-0.14714	-0.04426	0.36687	-0.34432	-0.57752	0.57051	0.21219
8	0.02825	0.02488	0.16909	0.03061	0.12996	-0.04803	-0.31963	0.92071

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Rotation Method: Varimax

Rotated Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	
FINCRAT8	-0.49284	0.63538	-0.07844	-0.03428	
WHTPR8	0.74590	-0.22161	0.07707	0.01171	pct of white pers in fam unrelind belw p
KIDS8	-0.25610	-0.43786	0.66861	0.05839	percentage of total pop under 6 years ol
OLD8	0.33289	0.10151	-0.26787	-0.14603	percentage of total pop over 64 years ol
CHILD8	-0.62348	-0.30052	0.45454	0.04867	percentage of total pop under 18 years o
BOOM8	0.65855	0.04945	0.31851	0.12499	percentage of persons born 1945-1960
MAGE18	-0.30970	0.14613	-0.62433	-0.03558	percentage of persons aged 45-54
MAGE28	-0.02840	0.04175	-0.68363	-0.14410	percentage of persons aged 55-64
RGRWTH18	0.74209	0.01281	0.34772	0.06228	pct of occ hsg units head moved 1979-198
RGRWTH28	0.01638	0.11251	0.75446	-0.00230	pct of occ hsg units head moved 1975-197

RGRWTH38	0.83612	-0.00370	0.11087	0.02068	pct of ohu rental head moved 1979-1980
RGRWTH48	0.82007	-0.01531	0.06126	-0.07170	pct of ohu rental head moved 1975-1978
HSG708	-0.23175	0.11949	0.74648	0.02089	pct of occ hsg units built in 70s
HSG608	-0.03530	0.02981	0.02307	0.10781	pct of occ hsg units built in 60s
HSG508	-0.29528	-0.06736	-0.60995	0.00318	pct of occ hsg units built in 50s
FNVMARX8	0.80088	0.19120	-0.10695	0.03563	pct of females 15 and over never married
FMRSPPX8	-0.89321	-0.02321	0.14688	0.06036	pct of fem15+ married spouse present
FDIVORX8	0.66937	-0.18986	0.03691	-0.02823	pct of females 15 and over divorced
WFHNK8	0.08955	-0.07192	-0.25654	0.06918	pct of white hh f headed fam wo ch 18
WFHWK8	0.20615	-0.31987	0.11477	0.06307	pct of white hh f headed fam w ch 18
MNVMARX8	0.80800	0.05206	-0.13215	0.01859	pct of males 15+ never married
MMRSPPX8	-0.93439	0.04682	0.10103	0.01588	pct of m 15+ married spouse present
MDIVORX8	0.74587	-0.18736	0.03306	-0.00228	pct of males 15 and over divorced
WMHNK8	0.13159	-0.04849	-0.05961	0.05447	pct of white hh m headed fam wo ch 18
WMHWK8	0.00272	-0.09225	0.14498	0.06675	pct of white hh m headed fam w ch 18
WMCNK8	-0.32149	0.20276	-0.29367	0.13075	pct of white hh mar couple fam wo ch 18
WMCWK8	-0.55778	0.01720	0.24465	0.20336	pct of white hh mar couple fam w ch 18
WHSG8	-0.26482	-0.47469	-0.07615	0.72999	Pct of whites 25+ high school graduates
WCOL8	0.04619	0.47368	0.11859	0.69688	Pct of whites 25+ some coll no degree
WCGR8	0.11901	0.82620	0.05570	0.34309	Pct of whites 25+ coll grad and more
WFEMPX8	0.06540	0.07975	0.01319	0.91190	percentage of white fem 16+ empl
WFUEMPX8	0.10795	-0.26299	0.14313	0.31860	percentage of white fem 16+ unempl
FNOPART8	0.06405	-0.09610	-0.00669	-0.22572	Pct of females aged 16+ worked <=26 week
WMEMPX8	-0.15406	0.12205	0.02641	0.94573	percentage of white males 16+ empl
WMUEMPX8	0.31145	-0.39910	0.04564	0.31621	percentage of white males 16+ unempl
MNOPART8	0.62936	-0.16662	-0.13602	-0.20679	Pct of males aged 16+ worked <=26 weeks
PROF8	0.06157	0.79047	0.09277	-0.00393	Professional tech workers as pct of tot
MGRS8	-0.23487	0.73679	0.02175	-0.03247	Managers administrators as pct of total
SALE8	-0.31220	0.64269	-0.09400	0.03364	Sales workers as pct of total
ADMN8	0.25023	-0.22890	-0.18868	0.10491	Admin support wkrs as pct of total
CRFT8	-0.35317	-0.68217	0.04148	0.03820	Craft workers as pct of total
OPER8	0.02979	-0.83326	0.07497	0.00931	Operatives as pct of total
SERV8	0.08146	-0.58672	-0.06479	-0.03830	Service workers as pct of total

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Rotation Method: Varimax

Rotated Factor Pattern

	FACTOR5	FACTOR6	FACTOR7	FACTOR8	
FINCRAT8	-0.35187	-0.07927	0.18196	0.09751	
WHTPR8	0.15055	0.04098	0.32084	-0.05065	pct of white pers in fam unrelind belw p
KIDS8	-0.05806	0.17295	-0.03657	-0.06077	percentage of total pop under 6 years ol
OLD8	0.71315	-0.06965	0.24105	0.34025	percentage of total pop over 64 years ol
CHILD8	-0.34943	0.15071	0.14965	-0.19020	percentage of total pop under 18 years o
BOOM8	0.08432	0.02188	-0.48693	-0.24586	percentage of persons born 1945-1960
MAGE18	-0.45767	-0.04576	0.12893	0.20426	percentage of persons aged 45-54
MAGE28	0.13678	-0.09576	0.08473	0.39471	percentage of persons aged 55-64
RGRWTH18	-0.12850	0.05658	-0.22804	0.07192	pct of occ hsg units head moved 1979-198
RGRWTH28	-0.13519	-0.07803	0.04705	0.12814	pct of occ hsg units head moved 1975-197
RGRWTH38	-0.02096	0.06081	-0.19434	-0.00440	pct of ohu rental head moved 1979-1980
RGRWTH48	0.12996	-0.01323	0.01800	0.06221	pct of ohu rental head moved 1975-1978
HSG708	-0.23619	-0.08874	-0.03095	0.17726	pct of occ hsg units built in 70s
HSG608	-0.49625	0.00723	-0.02657	0.05154	pct of occ hsg units built in 60s
HSG508	-0.19109	-0.02365	-0.16844	-0.00663	pct of occ hsg units built in 50s
FNVMARX8	-0.05697	-0.01003	-0.08241	-0.36724	pct of females 15 and over never married
FMRSPPX8	-0.27173	-0.08994	-0.04556	0.02743	pct of fem15+ married spouse present
FDIVORX8	0.04163	0.22522	-0.10082	0.25211	pct of females 15 and over divorced
WFHNK8	0.34370	0.37131	-0.03940	0.01587	pct of white hh f headed fam wo ch 18
WFHWK8	-0.11364	0.65064	-0.02652	0.03040	pct of white hh f headed fam w ch 18
MNVMARX8	0.05487	-0.04016	0.01693	-0.42807	pct of males 15+ never married
MMRSPPX8	-0.09815	-0.00715	-0.09329	0.14663	pct of m 15+ married spouse present
MDIVORX8	0.00895	0.07693	0.07325	0.33481	pct of males 15 and over divorced
WMHNK8	0.14257	0.40575	0.03088	-0.07989	pct of white hh m headed fam wo ch 18
WMHWK8	-0.10053	0.44612	-0.04354	-0.00460	pct of white hh m headed fam w ch 18
WMCNK8	0.07103	0.61647	0.01660	0.16099	pct of white hh mar couple fam wo ch 18
WMCWK8	-0.23638	0.44834	0.06101	-0.05056	pct of white hh mar couple fam w ch 18
WHSG8	-0.01178	0.10246	-0.06471	0.08256	Pct of whites 25+ high school graduates
WCOL8	-0.04830	0.02766	-0.08911	0.04121	Pct of whites 25+ some coll no degree
WCGR8	-0.06502	0.02787	0.09802	-0.18847	Pct of whites 25+ coll grad and more
WFEMPX8	-0.08408	0.13352	-0.27302	-0.06428	percentage of white fem 16+ empl
WFUEMPX8	-0.10662	0.13200	0.12691	-0.12874	percentage of white fem 16+ unempl
FNOPART8	0.37025	-0.02885	0.70589	0.08028	Pct of females aged 16+ worked <=26 week
WMEMPX8	-0.10543	0.10648	-0.04324	-0.00089	percentage of white males 16+ empl
WMUEMPX8	0.00344	0.17490	0.10068	-0.06233	percentage of white males 16+ unempl
MNOPART8	0.41031	0.02367	0.41570	0.04006	Pct of males aged 16+ worked <=26 weeks
PROF8	0.18068	-0.02262	0.01431	-0.27720	Professional tech workers as pct of tot
MGRS8	-0.28718	-0.06363	0.18299	0.25367	Managers administrators as pct of total
SALE8	-0.30371	-0.07603	-0.02237	0.18030	Sales workers as pct of total
ADMN8	0.25998	0.01717	-0.52594	0.11796	Admin support wkrs as pct of total

CRFT8	-0.14818	-0.04224	-0.05704	0.00409	Craft workers as pct of total
OPER8	-0.00184	0.06681	0.10274	0.01259	Operatives as pct of total
SERV8	0.00815	0.16161	0.09093	-0.02900	Service workers as pct of total

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Rotation Method: Varimax

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
9.385980	5.632462	3.774831	3.342174	2.359117	1.804864	1.771989	1.302641

Final Communality Estimates: Total = 29.374057

FINCRAT8	WHTPR8	KIDS8	OLD8	CHILD8	BOOM8	MAGE18	MAGE28	RGRWTH18
0.826635	0.741407	0.746065	0.901511	0.891415	0.858334	0.778216	0.681509	0.752542
RGRWTH28	**RGRWTH38**	**RGRWTH48**	**HSG708**	**HSG608**	**HSG508**	**FNVMARX8**	**FMRSPPX8**	**FDIVORX8**
0.625135	0.753755	0.702895	0.721691	0.263974	0.529273	0.835664	0.908333	0.612446
WFHNK8	**WFHWK8**	**MNVMARX8**	**MMRSPPX8**	**MDIVORX8**	**WMHNK8**	**WMHWK8**	**WMCNK8**	**WMCWK8**
0.341600	0.599843	0.861533	0.925622	0.715987	0.218486	0.245042	0.659079	0.675789
WHSG8	**WCOL8**	**WCGR8**	**WFEMPX8**	**WFUEMPX8**	**FNOPART8**	**WMEMPX8**	**WMUEMPX8**	**MNOPART8**
0.855787	0.738951	0.867718	0.945936	0.264280	0.706980	0.958060	0.402973	0.828451
PROF8	**MGRS8**	**SALE8**	**ADMN8**	**CRFT8**	**OPER8**	**SERV8**		
0.747462	0.783910	0.651518	0.520027	0.620284	0.716100	0.391839		

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Initial Factor Method: Principal Factors

Prior Communality Estimates: SMC

FINCRAT9	WHTPR9	KIDS9	OLD9	CHILD9	BOOM9	MAGE19	MAGE29	RGRWTH19
0.839174	0.882881	0.887051	0.930582	0.933695	0.758433	0.756187	0.827521	0.967098
RGRWTH29	**RGRWTH39**	**RGRWTH49**	**HSG709**	**HSG609**	**HSG509**	**FNVMARX9**	**FMRSPPX9**	**FDIVORX9**
0.676602	0.975657	0.878166	0.556593	0.503750	0.691882	0.962917	0.981627	0.821269
WFHNK9	**WFHWK9**	**MNVMARX9**	**MMRSPPX9**	**MDIVORX9**	**WMHNK9**	**WMHWK9**	**WMCNK9**	**WMCWK9**
0.438722	0.632064	0.980416	0.987212	0.868670	0.237921	0.561854	0.931647	0.946180
WHSG9	**WCOL9**	**WCGR9**	**WFEMPX9**	**WFUEMPX9**	**FNOPART9**	**WMEMPX9**	**WMUEMPX9**	**MNOPART9**
0.947634	0.763648	0.971349	0.874851	0.402877	0.256108	0.905143	0.450012	0.931748
PROF9	**MGRS9**	**SALE9**	**ADMN9**	**CRFT9**	**OPER9**	**SERV9**		
0.910641	0.870226	0.752796	0.772450	0.826303	0.869401	0.599193		

Eigenvalues of the Reduced Correlation Matrix: Total = 33.5201517 Average = 0.77953841

	1	2	3	4	5	6	7
Eigenvalue	11.6791	6.8298	4.3236	2.1197	1.4894	1.2687	1.1511
Difference	4.8493	2.5062	2.2040	0.6303	0.2207	0.1176	0.0916
Proportion	0.3484	0.2038	0.1290	0.0632	0.0444	0.0378	0.0343
Cumulative	0.3484	0.5522	0.6812	0.7444	0.7888	0.8267	0.8610
	8	**9**	**10**	**11**	**12**	**13**	**14**
Eigenvalue	1.0595	0.8910	0.6748	0.5195	0.4435	0.3111	0.2744
Difference	0.1685	0.2162	0.1552	0.0760	0.1324	0.0368	0.0284
Proportion	0.0316	0.0266	0.0201	0.0155	0.0132	0.0093	0.0082
Cumulative	0.8926	0.9192	0.9393	0.9548	0.9681	0.9773	0.9855
	15	**16**	**17**	**18**	**19**	**20**	**21**
Eigenvalue	0.2460	0.2340	0.1865	0.1635	0.1351	0.1184	0.0690
Difference	0.0120	0.0475	0.0231	0.0284	0.0166	0.0494	0.0065
Proportion	0.0073	0.0070	0.0056	0.0049	0.0040	0.0035	0.0021
Cumulative	0.9929	0.9999	1.0054	1.0103	1.0143	1.0179	1.0199
	22	**23**	**24**	**25**	**26**	**27**	**28**
Eigenvalue	0.0625	0.0606	0.0411	0.0236	0.0127	0.0042	0.0014
Difference	0.0019	0.0195	0.0176	0.0108	0.0086	0.0027	0.0037
Proportion	0.0019	0.0018	0.0012	0.0007	0.0004	0.0001	0.0000
Cumulative	1.0218	1.0236	1.0248	1.0255	1.0259	1.0260	1.0261
	29	**30**	**31**	**32**	**33**	**34**	**35**
Eigenvalue	-0.0023	-0.0067	-0.0169	-0.0229	-0.0251	-0.0357	-0.0375
Difference	0.0044	0.0102	0.0061	0.0022	0.0106	0.0018	0.0104

Proportion	-0.0001	-0.0002	-0.0005	-0.0007	-0.0007	-0.0011	-0.0011
Cumulative	1.0260	1.0258	1.0253	1.0246	1.0239	1.0228	1.0217

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Initial Factor Method: Principal Factors

	36	37	38	39	40	41	42
Eigenvalue	-0.0479	-0.0540	-0.0615	-0.0719	-0.0909	-0.1190	-0.1356
Difference	0.0061	0.0075	0.0105	0.0190	0.0281	0.0166	0.0103
Proportion	-0.0014	-0.0016	-0.0018	-0.0021	-0.0027	-0.0035	-0.0040
Cumulative	1.0203	1.0186	1.0168	1.0147	1.0119	1.0084	1.0044

	43
Eigenvalue	-0.1459
Difference	
Proportion	-0.0044
Cumulative	1.0000

8 factors will be retained by the NFACTOR criterion.

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Initial Factor Method: Principal Factors

Scree Plot of Eigenvalues

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Initial Factor Method: Principal Factors

Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	
FINCRAT9	-0.70257	-0.49697	-0.02296	-0.13755	
WHTPR9	0.83363	-0.03483	0.06494	-0.23812	pct of white pers in fam unrelind belw p
KIDS9	-0.00935	0.60999	0.40976	-0.46241	percentage of tot pop under 6 years old
OLD9	0.21806	-0.20792	-0.76575	-0.09761	percentage of tot pop over 64 years old
CHILD9	-0.24125	0.66439	0.37717	-0.41834	percentage of tot pop under 18 years old
BOOM9	-0.33087	-0.09965	0.55915	-0.00268	percentage of persons born 1945-1960
MAGE19	-0.62769	-0.08487	-0.14820	0.09485	percentage of persons aged 45-54
MAGE29	-0.25449	-0.07809	-0.78559	0.10830	percentage of persons aged 55-64
RGRWTH19	0.71467	-0.28697	0.38533	0.20311	pct of occ hsg units head moved 1989-199
RGRWTH29	-0.06365	-0.14721	0.52775	-0.23284	pct of occ hsg units head moved 1985-198
RGRWTH39	0.79909	-0.29794	0.22867	0.22618	pct of ohu rental head moved 1989-1990
RGRWTH49	0.79118	-0.28795	0.03967	0.01704	pct of ohu rental head moved 1985-1989
HSG709	-0.23117	0.05557	0.32370	0.08257	pct of occ hsg units built in 70s
HSG609	-0.08559	0.05355	-0.22587	0.29464	pct of occ hsg units built in 60s
HSG509	-0.19200	0.15437	-0.66031	0.16635	pct of occ hsg units built in 50s
FNVMARX9	0.74039	-0.43556	0.21861	0.20092	pct of femal 15 and over never married
FMRSPPX9	-0.92541	0.29577	0.03761	-0.06238	pct of fem15+ married spouse present
FDIVORX9	0.60150	0.07188	0.04116	0.12020	pct of females 15 and over divorced
WFHNK9	0.28641	0.21397	-0.27664	-0.03533	pct of white hh f headed fam wo ch 18
WFHWK9	0.38450	0.47627	0.16143	-0.14307	pct of white hh f headed fam w ch 18
MNVMARX9	0.83077	-0.31120	0.17185	0.10906	pct of males 15+ never married
MMRSPPX9	-0.92294	0.23896	-0.10016	-0.11880	pct of m 15+ married spouse present
MDIVORX9	0.69745	0.05104	0.04482	0.10385	pct of males 15 and over divorced
WMHNK9	0.16917	0.18839	-0.06574	0.03718	pct of white hh m headed fam wo ch 18
WMHWK9	0.03535	0.21774	0.13194	0.11655	pct of white hh m headed fam w ch 18
WMCNK9	-0.81831	0.08628	-0.43880	0.04704	pct of white hh mar couple fam wo ch 18
WMCWK9	-0.79137	0.36175	0.34774	-0.08195	pct of white hh mar couple fam w ch 18
WHSG9	0.02129	0.86370	-0.16629	0.26605	Pct of whites 25+ high school graduates
WCOL9	-0.36974	0.07529	0.27147	0.30627	Pct of whites 25+ some coll no degree
WCGR9	-0.24375	-0.89705	0.14482	-0.18796	Pct of whites 25+ coll grad and more
WFEMPX9	-0.42299	-0.04403	0.52646	0.56952	percentage of white fem 16+ empl
WFUEMPX9	0.34829	0.13160	0.21660	-0.07722	percentage of white fem 16+ unempl
FNOPART9	0.15285	0.02846	-0.06514	-0.40931	Pct of females 16+ wrk <=26 weeks in 198
WMEMPX9	-0.63334	-0.03199	0.48631	0.26419	percentage of white males 16+ empl
WMUEMPX9	0.42060	0.24732	0.04827	-0.05268	percentage of white males 16+ unempl
MNOPART9	0.75701	0.02816	-0.36911	-0.30762	Pct of males 16+ wrk <=26 weeks in 1989
PROF9	-0.21920	-0.75993	0.07007	-0.19694	Professional tech workers as pct of tot
MGRS9	-0.54020	-0.63967	0.01111	-0.09649	Managers administrators as pct of total
SALE9	-0.48784	-0.50557	-0.05736	-0.01626	Sales workers as pct of total
ADMN9	0.19131	0.25371	-0.14618	0.50878	Admin support wkrs as pct of total
CRFT9	-0.20463	0.76820	0.10099	0.18855	Craft workers as pct of total
OPER9	0.38063	0.78397	-0.04532	-0.07680	Operatives as pct of total
SERV9	0.34157	0.55915	0.00193	-0.04458	Service workers as pct of total

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Initial Factor Method: Principal Factors

Factor Pattern

	FACTOR5	FACTOR6	FACTOR7	FACTOR8	
FINCRAT9	0.15888	0.10660	-0.06069	-0.03683	
WHTPR9	0.22725	-0.05494	-0.16099	-0.02361	pct of white pers in fam unrelind belw p
KIDS9	-0.03137	0.05176	0.04343	0.18163	percentage of tot pop under 6 years old
OLD9	-0.14242	-0.04637	0.36972	-0.16864	percentage of tot pop over 64 years old
CHILD9	0.09994	-0.00737	-0.03721	0.08230	percentage of tot pop under 18 years old
BOOM9	-0.34281	0.22035	0.24817	-0.10124	percentage of persons born 1945-1960
MAGE19	0.35125	0.19931	-0.24054	-0.16161	percentage of persons aged 45-54
MAGE29	0.22539	0.17363	0.00290	0.15433	percentage of persons aged 55-64
RGRWTH19	0.15895	0.06098	-0.00736	0.00794	pct of occ hsg units head moved 1989-199
RGRWTH29	0.12447	-0.18778	0.36747	-0.04095	pct of occ hsg units head moved 1985-198
RGRWTH39	0.16747	0.05543	-0.06305	-0.01673	pct of ohu rental head moved 1989-1990
RGRWTH49	0.20038	-0.12157	0.13141	-0.03840	pct of ohu rental head moved 1985-1989
HSG709	0.52447	-0.20959	0.13606	-0.12594	pct of occ hsg units built in 70s
HSG609	0.47342	-0.01873	-0.12121	-0.04728	pct of occ hsg units built in 60s
HSG509	-0.08970	0.04570	-0.06982	0.25330	pct of occ hsg units built in 50s
FNVMARX9	-0.08616	-0.18496	-0.21984	0.07055	pct of femal 15 and over never married
FMRSPPX9	0.04096	-0.03282	-0.00557	-0.00774	pct of fem15+ married spouse present
FDIVORX9	0.05316	0.55965	0.25787	-0.01209	pct of females 15 and over divorced
WFHNK9	-0.28104	0.22992	0.01440	0.23579	pct of white hh f headed fam wo ch 18
WFHWK9	0.12948	0.22788	0.00803	0.19097	pct of white hh f headed fam w ch 18
MNVMARX9	-0.05176	-0.08429	-0.28193	0.07751	pct of males 15+ never married
MMRSPPX9	0.00163	-0.00549	0.05731	-0.00661	pct of m 15+ married spouse present
MDIVORX9	0.10369	0.37789	0.30144	-0.08822	pct of males 15 and over divorced
WMHNK9	-0.24252	-0.01285	-0.09466	0.14004	pct of white hh m headed fam wo ch 18
WMHWK9	-0.05722	0.32725	-0.09652	-0.47403	pct of white hh m headed fam w ch 18
WMCNK9	0.02931	0.06195	-0.13633	0.06132	pct of white hh mar couple fam wo ch 18
WMCWK9	0.04183	-0.18039	-0.03238	-0.06403	pct of white hh mar couple fam w ch 18
WHSG9	0.00029	-0.05828	-0.00918	-0.23196	Pct of whites 25+ high school graduates

WCOL9	0.17253	-0.03659	0.30266	0.49026	Pct of whites 25+ some coll no degree
WCGR9	-0.08445	0.01781	-0.12449	0.07391	Pct of whites 25+ coll grad and more
WFEMPX9	-0.18368	-0.03341	-0.05391	0.16604	percentage of white fem 16+ empl
WFUEMPX9	0.15561	0.15770	-0.15764	0.06384	percentage of white fem 16+ unempl
FNOPART9	0.08575	-0.02766	0.05796	-0.03181	Pct of females 16+ wrk <=26 weeks in 198
WMEMPX9	-0.07571	0.22910	-0.12451	0.04840	percentage of white males 16+ empl
WMUEMPX9	0.06080	0.10315	-0.15286	0.18746	percentage of white males 16+ unempl
MNOPART9	0.06085	-0.15886	0.03990	0.02512	Pct of males 16+ wrk <=26 weeks in 1989
PROF9	-0.25593	0.05339	-0.12010	-0.14290	Professional tech workers as pct of tot
MGRS9	0.16477	0.16155	0.17057	0.09146	Managers administrators as pct of total
SALE9	0.25876	0.11433	0.04938	0.17770	Sales workers as pct of total
ADMN9	-0.09795	-0.34217	0.31037	-0.02844	Admin support wkrs as pct of total
CRFT9	0.00762	0.01627	-0.08319	-0.18598	Craft workers as pct of total
OPER9	0.05386	0.03770	-0.05421	0.07641	Operatives as pct of total
SERV9	-0.00509	0.00463	-0.10027	0.09044	Service workers as pct of total

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Initial Factor Method: Principal Factors

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
11.679147	6.829821	4.323640	2.119659	1.489379	1.268721	1.151078	1.059492

Final Communality Estimates: Total = 29.920937

FINCRAT9	WHTPR9	KIDS9	OLD9	CHILD9	BOOM9	MAGE19	MAGE29	RGRWTH19
0.801681	0.838203	0.792440	0.874252	0.835085	0.669983	0.679236	0.804513	0.811943
RGRWTH29	RGRWTH39	RGRWTH49	HSG709	HSG609	HSG509	FNVMARX9	FMRSPPX9	FDIVORX9
0.545928	0.866143	0.784421	0.521502	0.389427	0.603546	0.920990	0.952016	0.765786
WFHNK9	WFHWK9	MNVMARX9	MMRSPPX9	MDIVORX9	WMHNK9	WMHWK9	WMCNK9	WMCWK9
0.393244	0.526439	0.923734	0.936424	0.754043	0.157367	0.424040	0.898871	0.924205
WHSG9	WCOL9	WCGR9	WFEMPX9	WFUEMPX9	FNOPART9	WMEMPX9	WMUEMPX9	MNOPART9
0.902150	0.672937	0.948828	0.847696	0.269515	0.208445	0.784502	0.316018	0.835889
PROF9	MGRS9	SALE9	ADMN9	CRFT9	OPER9	SERV9		
0.772435	0.801130	0.611193	0.605007	0.719583	0.780550	0.449598		

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Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2	3	4	5	6	7	8
1	0.86099	-0.30895	-0.32952	-0.05570	-0.17787	0.01003	0.13832	0.02617
2	-0.34334	-0.92779	0.00976	-0.03434	-0.02628	-0.12479	0.04081	0.04602
3	0.23668	-0.02323	0.51594	-0.63590	0.38968	-0.34546	-0.03102	0.02681
4	0.22604	-0.15226	0.71042	0.40569	0.01867	0.49616	0.08778	0.04702
5	0.09651	-0.03947	-0.15736	0.51335	0.75059	-0.29160	0.16025	-0.16353
6	-0.06558	0.13584	0.15955	0.13219	-0.28483	-0.41929	0.74158	0.35698
7	-0.13746	-0.00158	-0.16675	-0.36925	0.22994	0.51433	0.62647	-0.32723
8	0.03444	-0.00361	0.20594	0.09701	-0.34364	-0.30568	0.04943	-0.85616

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Rotation Method: Varimax

Rotated Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	
FINCRAT9	-0.45539	0.70806	0.11164	0.12948	
WHTPR9	0.73810	-0.20658	-0.43324	-0.01658	pct of white pers in fam unrelind belw p
KIDS9	-0.23116	-0.49462	-0.06470	-0.47627	percentage of tot pop under 6 years old
OLD9	-0.01150	0.15753	-0.61968	0.21022	percentage of tot pop over 64 years old
CHILD9	-0.42304	-0.49213	-0.01038	-0.34688	percentage of tot pop under 18 years old
BOOM9	-0.20407	0.22554	0.42152	-0.58312	percentage of persons born 1945-1960
MAGE19	-0.47660	0.27584	0.18028	0.45040	percentage of persons aged 45-54
MAGE29	-0.33847	0.16696	-0.22174	0.71291	percentage of persons aged 55-64
RGRWTH19	0.86359	0.00756	0.09238	-0.09944	pct of occ hsg units head moved 1989-199
RGRWTH29	0.04043	0.14859	0.00715	-0.52205	pct of occ hsg units head moved 1985-198
RGRWTH39	0.91617	-0.00913	0.00200	0.02703	pct of ohu rental head moved 1989-1990
RGRWTH49	0.80123	-0.00528	-0.31169	-0.01795	pct of ohu rental head moved 1985-1989
HSG709	-0.08152	-0.04916	0.13779	0.01769	pct of occ hsg units built in 70s

HSG609	-0.01699	-0.08372	0.05449	0.54682	pct of occ hsg units built in 60s
HSG509	-0.33033	-0.08495	-0.07252	0.50312	pct of occ hsg units built in 50s
FNVMARX9	0.92064	0.11806	0.04254	-0.06444	pct of femal 15 and over never married
FMRSPPX9	-0.89691	0.01408	0.27056	0.01016	pct of fem15+ married spouse present
FDIVORX9	0.46268	-0.19822	-0.05544	-0.00850	pct of females 15 and over divorced
WFHNK9	0.06361	-0.23375	-0.13305	0.04197	pct of white hh f headed fam wo ch 18
WFHWK9	0.17642	-0.51750	-0.08643	-0.08631	pct of white hh f headed fam w ch 18
MNVMARX9	0.92942	0.00221	-0.05297	-0.02671	pct of males 15+ never married
MMRSPPX9	-0.93484	0.08298	0.15833	0.03700	pct of m 15+ married spouse present
MDIVORX9	0.55781	-0.23261	-0.15688	-0.04365	pct of males 15 and over divorced
WMHNK9	0.06909	-0.22371	0.01932	-0.03666	pct of white hh m headed fam wo ch 18
WMHWK9	-0.01679	-0.18517	0.12104	-0.04253	pct of white hh m headed fam w ch 18
WMCNK9	-0.80778	0.18305	0.11814	0.42026	pct of white hh mar couple fam wo ch 18
WMCWK9	-0.72367	-0.11260	0.34234	-0.21935	pct of white hh mar couple fam w ch 18
WHSG9	-0.26031	-0.85163	0.04904	0.15616	Pct of whites 25+ high school graduates
WCOL9	-0.21638	-0.02259	0.49772	-0.01083	Pct of whites 25+ some coll no degree
WCGR9	0.10026	0.93852	0.06487	-0.11182	Pct of whites 25+ coll grad and more
WFEMPX9	-0.09814	0.07478	0.88193	-0.14135	percentage of white fem 16+ empl
WFUEMPX9	0.31705	-0.20768	-0.01649	-0.02785	percentage of white fem 16+ unempl
FNOPART9	0.01492	-0.01691	-0.40861	-0.11825	Pct of females 16+ wrk <=26 weeks in 198
WMEMPX9	-0.36304	0.20796	0.72617	-0.12360	percentage of white males 16+ empl
WMUEMPX9	0.30331	-0.34133	-0.07772	0.03549	percentage of white males 16+ unempl
MNOPART9	0.49689	-0.22873	-0.69455	0.06473	Pct of males 16+ wrk <=26 weeks in 1989
PROF9	0.02764	0.81920	0.00045	-0.17999	Professional tech workers as pct of tot
MGRS9	-0.27966	0.78965	0.09919	0.05768	Managers administrators as pct of total
SALE9	-0.24689	0.62819	0.12056	0.22137	Sales workers as pct of total
ADMN9	0.12735	-0.41157	0.12867	0.06712	Admin support wkrs as pct of total
CRFT9	-0.36872	-0.67785	0.23795	0.01603	Craft workers as pct of total
OPER9	0.04327	-0.82941	-0.17341	0.00960	Operatives as pct of total
SERV9	0.10860	-0.61690	-0.10089	-0.01374	Service workers as pct of total

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Rotation Method: Varimax

Rotated Factor Pattern

	FACTOR5	FACTOR6	FACTOR7	FACTOR8	
FINCRAT9	0.21410	-0.11633	-0.06416	0.01513	
WHTPR9	0.03081	-0.24669	-0.01537	0.02687	pct of white pers in fam unrelind belw p
KIDS9	0.04596	-0.49294	0.03984	-0.12904	percentage of tot pop under 6 years old
OLD9	-0.28427	0.54692	0.20293	0.00115	percentage of tot pop over 64 years old
CHILD9	0.20489	-0.49354	-0.06337	-0.06256	percentage of tot pop under 18 years old
BOOM9	0.05109	-0.01922	0.19153	0.14181	percentage of persons born 1945-1960
MAGE19	0.26500	-0.15776	-0.03195	0.21094	percentage of persons aged 45-54
MAGE29	-0.18943	0.14810	0.16981	-0.13418	percentage of persons aged 55-64
RGRWTH19	0.13189	-0.06749	0.15949	0.01676	pct of occ hsg units head moved 1989-199
RGRWTH29	0.46198	-0.03616	0.05726	-0.17782	pct of occ hsg units head moved 1985-198
RGRWTH39	0.06019	-0.02097	0.13875	0.05132	pct of ohu rental head moved 1989-1990
RGRWTH49	0.11105	0.11049	0.12033	-0.07697	pct of ohu rental head moved 1985-1989
HSG709	0.69526	-0.03670	-0.02487	-0.08822	pct of occ hsg units built in 70s
HSG609	0.28035	0.03859	0.00692	0.00406	pct of occ hsg units built in 60s
HSG509	-0.40755	0.18312	0.00312	-0.17084	pct of occ hsg units built in 50s
FNVMARX9	-0.11809	0.05398	-0.18971	-0.02576	pct of femal 15 and over never married
FMRSPPX9	0.21180	-0.08882	-0.14422	-0.02249	pct of fem15+ married spouse present
FDIVORX9	-0.14665	-0.07135	0.67990	0.14286	pct of females 15 and over divorced
WFHNK9	-0.51918	-0.02491	0.19997	-0.07029	pct of white hh f headed fam wo ch 18
WFHWK9	-0.05217	-0.36988	0.25927	-0.07637	pct of white hh f headed fam w ch 18
MNVMARX9	-0.17689	-0.07634	-0.13713	0.02142	pct of males 15+ never married
MMRSPPX9	0.13487	-0.03010	-0.09346	-0.03675	pct of m 15+ married spouse present
MDIVORX9	-0.03617	0.03000	0.58762	0.12151	pct of males 15 and over divorced
WMHNK9	-0.30823	0.00396	-0.06438	-0.04076	pct of white hh m headed fam wo ch 18
WMHWK9	0.04612	-0.03984	0.16953	0.58357	pct of white hh m headed fam w ch 18
WMCNK9	-0.07490	0.03256	-0.12367	-0.01756	pct of white hh mar couple fam wo ch 18
WMCWK9	0.36257	-0.14752	-0.26320	-0.00441	pct of white hh mar couple fam w ch 18
WHSG9	0.00810	0.17242	0.00632	0.22910	Pct of whites 25+ high school graduates
WCOL9	0.21634	0.01590	0.18475	-0.54459	Pct of whites 25+ some coll no degree
WCGR9	-0.00263	-0.10325	-0.16597	-0.05499	Pct of whites 25+ coll grad and more
WFEMPX9	0.09437	0.09104	-0.10642	-0.07860	percentage of white fem 16+ empl
WFUEMPX9	0.03124	-0.33816	0.08634	0.04512	percentage of white fem 16+ unempl
FNOPART9	0.03554	-0.15647	0.01636	-0.03131	Pct of females 16+ wrk <=26 weeks in 198
WMEMPX9	0.14059	-0.19210	0.00136	0.10089	percentage of white males 16+ empl
WMUEMPX9	-0.14679	-0.26636	0.06189	-0.06239	percentage of white males 16+ unempl
MNOPART9	-0.19350	0.04068	0.00848	-0.10448	Pct of males 16+ wrk <=26 weeks in 1989
PROF9	-0.10323	0.00487	-0.16452	0.17447	Professional tech workers as pct of tot
MGRS9	0.20087	-0.03332	0.14794	-0.15120	Managers administrators as pct of total
SALE9	0.18935	-0.08237	0.07821	-0.20813	Sales workers as pct of total
ADMN9	0.01692	0.61355	0.00960	-0.14666	Admin support wkrs as pct of total
CRFT9	0.10496	-0.03423	-0.03155	0.23258	Craft workers as pct of total
OPER9	-0.11643	-0.19921	0.08571	-0.00182	Operatives as pct of total
SERV9	-0.13480	-0.16881	0.01037	-0.00952	Service workers as pct of total

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Rotation Method: Varimax

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
9.855809	7.071113	3.635601	2.723097	2.159505	1.898527	1.445646	1.131637

Final Communality Estimates: Total = 29.920937

FINCRAT9	WHTPR9	KIDS9	OLD9	CHILD9	BOOM9	MAGE19	MAGE29	RGRWTH19
0.801681	0.838203	0.792440	0.874252	0.835085	0.669983	0.679236	0.804513	0.811943

RGRWTH29	RGRWTH39	RGRWTH49	HSG709	HSG609	HSG509	FNVMARX9	FMRSPPX9	FDIVORX9
0.545928	0.866143	0.784421	0.521502	0.389427	0.603546	0.920990	0.952016	0.765786

WFHNK9	WFHWK9	MNVMARX9	MMRSPPX9	MDIVORX9	WMHNK9	WMHWK9	WMCNK9	WMCWK9
0.393244	0.526439	0.923734	0.936424	0.754043	0.157367	0.424040	0.898871	0.924205

WHSG9	WCOL9	WCGR9	WFEMPX9	WFUEMPX9	FNOPART9	WMEMPX9	WMUEMPX9	MNOPART9
0.902150	0.672937	0.948828	0.847696	0.269515	0.208445	0.784502	0.316018	0.835889

PROF9	MGRS9	SALE9	ADMN9	CRFT9	OPER9	SERV9
0.772435	0.801130	0.611193	0.605007	0.719583	0.780550	0.449598