Extreme avalanche runout: a comparison of empirical models

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Abstract: The prediction of snow avalanche runout distances and the probability of exceeding the predicted positions is the first and most important step required before making decisions about placement of facilities or control structures in snow avalanche prone terrain. There are two main prediction methods for calculating runout distances: (1) procedures linked to the selection of friction coefficients in avalanche dynamic models, and (2) empirical, statistical prediction based on terrain parameters for a set of extreme runout distances for the mountain range in question. Within the second method there are presently two competing empirical approaches to prediction: (*i*) ordinary least squares regression analysis related to angles measured for the path profile in question, and (*ii*) extreme value prediction of runout based on a Gumbel distribution of a dimensionless terrain parameter. In this paper a comparison of the two empirical methods with emphasis on the slope steepness in the runout zone is provided. The comparison is important, since the choice of method does affect the probability of the runout position being exceeded, particularly far into the runout zone where facilities are most likely to be located. The analysis shows that comparison of the models hinges on slope steepness in the runout zone and differences in calculating exceedance probabilities from the distributions used in the analysis (Gumbel distribution and Gaussian). The method based on the dimensionless Gumbel parameter provides more conservative predictions for flat terrain in the runout zone, and the regression – least squares method is more conservative for steep terrain in the runout zone. In addition, the Gumbel method is shown to be compatible with the characteristics of runout zone slope steepness shown by field data: there is very little dependence of runout distance on runout zone slope steepness.

Key words: snow avalanche, runout, empirical methods, statistical methods.

Résumé : La prédiction des distances de parcours d’une avalanche de neige et la probabilité de dépasser les positions prédites est la première et plus importante étape avant de prendre des décisions sur l’emplacement des structures d’équipement et de contrôle dans les secteurs sujets aux avalanches de neige. Il existe deux méthodes principales de prédiction pour les distances de parcours: (1) des procédures liées à la sélection des coefficients de frottement dans les modèles dynamiques d’avalanches, et (2) une prédiction empirique et statistique basée sur les paramètres de terrain pour un ensemble de distances de parcours extrêmes pour la chaîne de montagnes concernée. Dans la seconde méthode, il y a présentement deux approches empiriques concurrentes de prédiction: (*i*) une analyse ordinaire de régression par les moindres carrés en rapport avec les angles mesurés pour le profil de parcours en question, et (*ii*) la valeur extrême de prédiction du parcours basée sur la distribution de Gumbel d’un paramètre sans dimension de terrain. Dans cet article, on fournit une comparaison des deux méthodes empiriques avec une emphase sur la raideur de la pente dans la zone de parcours. La comparaison est importante puisque le choix de la méthode affecte la probabilité que la position de la déposition soit excédée, particulièrement loin dans la zone de parcours où les équipements sont le plus sujets à être localisés. L’analyse montre que la comparaison des modèles tourne autour de la raideur de la pente dans la zone de parcours et des différences dans le calcul des probabilités de dépassement en partant des distributions utilisées dans l’analyse (distribution de Gumbel et Gaussienne). La méthode basée sur le paramètre sans dimension de Gumbel fournit des prédictions plus conservatrices pour un terrain plat dans la zone de parcours et la méthode de régression par les moindres carrés est plus conservatrice pour les terrains abrupts dans la zone de parcours. De plus, on montre que la méthode de Gumbel est compatible avec les caractéristiques de raideur de la pente dans la zone de parcours comme le montrent les données de terrain: il y a une très faible dépendance de la distance de parcours par rapport à la raideur de la pente dans la zone de parcours.

Mots clés : avalanche de neige, parcours, méthodes empiriques, méthodes statistiques.

[Traduit par la Rédaction]

Introduction

Specification of expected runout distances and return periods in the runout zone is the first and most important step required for zoning in snow avalanche prone terrain. The traditional method for more than 40 years since the work of Voellmy (1955) has been to select friction coefficients in a

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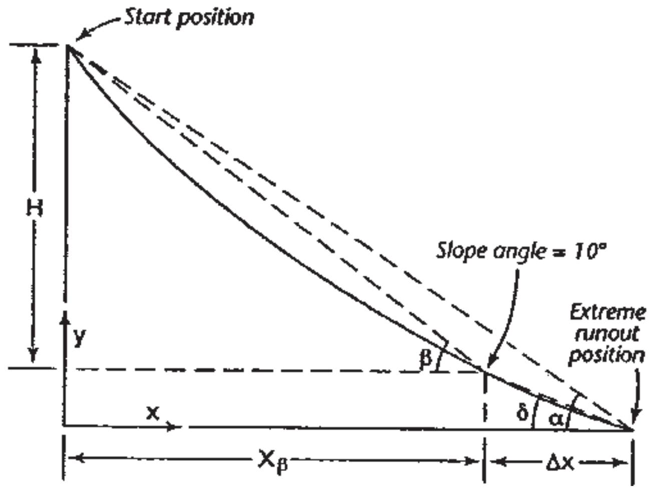
Fig. 1. Geometry for regression (α, β) and runout ratio (RR) calculations.



dynamics model in a deterministic sense or with return periods attached to input friction coefficients related to expected fracture heights (e.g., Salm 1993, 1997). The work of Bovis and Mears (1976) and Lied and Bakkehøi (1980) introduced an entirely new method. With their method extreme runout (variable return periods: 50–300 years) can be predicted from topographic terrain parameters for a set of avalanche paths in a mountain range from probability and statistics with the uncertainty factor being quantified statistically. This method is now used in many countries and jurisdictions in the world, but the dynamics method is still very popular.

Within the empirical method to predict extreme runout, there are two related methods used to enable statistical prediction of runout (Jóhannesson 1998). The more popular was introduced by Lied and Bakkehøi (1980) based on a multiple least squares analysis of a response variable angle α (see Fig.1), which represents the extreme runout position with other slope angles as predictor variables. This method is hereafter called the regression or least squares method. To apply the regression method to estimate runout distances probabilistically, it is necessary to make an assumption about the probability density function of the residuals. In this paper I have chosen examples where the probability density function of residuals is Gaussian.

An alternative, but related method, was proposed by McClung and Lied (1987) and expanded upon by McClung and Mears (1991). This second method involves fitting a set of extreme runout positions to a Gumbel distribution using a dimensionless quantity called the runout ratio (RR), i.e., $\Delta x/X_\beta$, which is a ratio of two length scales (depicted in Fig. 1 and explained in more detail in the next section). Application of the method implies that the runout ratio obeys a double exponential (Gumbel) distribution in the runout zone.

Model comparison is the focus of this paper, and I illustrate the differences as a function of slope steepness and estimates of predictions of exceedance probability in the runout zone. Slope steepness in the runout zone is implicitly contained in both the Gumbel method and the regression method as they are applied in practice, but the predicted behaviour on runout zone steepness is entirely different for the two approaches. In this paper I show that the regression method predicts a strong dependence of runout distance on runout zone slope, whereas the Gumbel method does not display an explicit dependence on the runout zone slope angle. Data from extreme avalanche runout are shown to imply a weak correlation with the runout zone slope, in better agreement with the Gumbel method. Probability distributions used to calculate runout also differ for the two models and affect predictions. The choice of an empirical method must be made with an awareness of these differences in predictions.

Extreme runout: empirical statistical models

In this section I present basic explanations of the two main methods. Following this section, I give a description of the data used in support of the analysis.

Regression method using terrain variables

The first comprehensive attempt at modelling extreme runout using terrain variables for snow avalanches was made by Lied and Bakkehøi (1980). They established a reference point from which to measure runout (called the β point), with β defined as the angle sighting from the position that the slope first declines to 10° to the start position (Fig. 1). The procedure was to relate α (where α is the angle obtained by sighting from the extreme runout position (return periods varying from 50 to 300 years) to the start position) to other topographic parameters including slope angles, vertical drop *H*, and the second derivative of terrain profile, *y*, where $y(x)$ is the profile function. This was done using a standard, least squares analysis. In this way, they developed a runout model relating a response variable, α, to β and possibly other parameters.

The original paper was a landmark: extreme runout can be related to terrain parameters without using avalanche dynamics. The method outlined by Lied and Bakkehøi (1980) has now been used in many countries for specifying extreme runout calculated from the historical record in a given mountain range. The method allows the use of empirical equations to predict the probability of extreme runout for locations where runout is unknown.

Subsequent analysis has shown that, for Norwegian data, the only significant predictor variable from those originally proposed is β. McClung and Mears (1991) analyzed data from five mountain ranges, and they showed the form of the regression equation to be

$$[1] \qquad \alpha = C_0 + C_1 \beta$$

where in most cases (including data from Norway) the first constant (C_0) is not significant. Jóhannesson (1998) showed that data from Iceland are also described by eq. [1] with $C_0 = 0$. From eq. [1], the extreme point is determined by relationships between random variations of α and β. In a later section I will explain how eq. [1] is used to calculate runout distances for a set of exceedance probabilities in the runout zone.

Gumbel distribution applied to runout

An alternate topographical method was proposed by McClung and Lied (1987) and extended and expanded upon by McClung et al. (1989), McClung and Mears (1991), and Nixon and McClung (1993). They showed, based on extreme runout information collected from more than 500 avalanche paths in six mountain ranges, that the runout ratio(RR), $\Delta x/X_\beta$, (see Fig. 1 for definitions of Δx and X_β) obeys a Type I extreme value (or Gumbel) distribution. The runout ratio (RR) is the ratio of the horizontal reach, Δx, (extreme runout distance from the β point) to X_β, i.e., the horizontal reach from the start position to the β point (see Fig. 1). The idea is appealing because a set of extreme values (runout) might be expected to follow an extreme value distribution. Further, McClung and Mears (1991) showed the necessity of using data from a given mountain range to specify runout ratios in that range, i.e., there are considerable differences between Gumbel parameters from one mountain range to another. The RR is calculated from three slope angles: α, β, and the slope steepness in the runout zone defined as δ, the angle obtained by sighting from the extreme runout position (the α point) to the β point. In fact, calculation of the RR is very sensitive to δ, given values of α and β. In terms of α, β, and δ the runout ratio is

$$\text{[2]} \qquad \mathrm{RR} = \frac{\Delta x}{X_\beta} = \frac{\tan\beta - \tan\alpha}{\tan\alpha - \tan\delta}$$

Data used in analysis

The data used to support the analysis in this paper represent high quality information collected over more than 20 years on extreme runout in Norway, the United States, and Canada. Here I provide a brief summary of the techniques used and data accuracy. The focus is on the estimation of the angles: α, β, and δ. For all data sets, there is no return period estimate available for the extreme runout position: the extreme runout positions for a set of avalanche paths are found and they have variable return periods in the range of 50–300 years (with most exceeding 100 years). Thus, no explicit information on return periods should be attached to any of the data cited in this paper. McClung (2000) has provided a method to estimate the distribution of return periods in the runout zone using extreme avalanche data. The data represent extreme, long return period events and were collected in the field during the summer rather than for particular avalanche events observed in winter.

Data from Canada

The data from Canada (McClung et al. 1989; McClung and Mears 1991; Nixon and McClung 1993) are from the British Columbia Coast Mountains (western B.C.), the Canadian Rockies, and the Purcell Mountain range (eastern B.C. and Alberta).

For these data, the positions of α and β were all found in the field, and start positions were all estimated by field inspection. The angle δ was determined by sighting between the positions of α and β with a clinometer. In addition, the entire slope profile between the positions of α and β was measured in the field by measuring distances and angles for typically about 10 segments. These measurements gave accurate estimates of Δx. The values of X_β were determined from maps or calculations from measured angles in combination with maps, depending on which produced the best accuracy. The angles α and β were measured in the field using a clinometer and also estimated on maps for consistency. The position of α was determined by examining vegetation damage in the runout zone, usually by locating or coring trees (usually several hundred years old) that demarcate the extreme position. The maximum accuracy of angles measured in the field is about 0.5° using a clinometer. Lengths (Δx) measured and estimated in the runout zone are correct to better than 20 m, and RR is correct to at least two significant figures. Map scales used were all larger than 1 : 50 000. Field checking showed that map scales of 1 : 50 000 are too small to achieve the desired accuracy in the runout zone. Avalanche paths were used in the data set only if there was clear evidence that the extreme reach position was located in the field and if the top of the start zone could be identified from field inspection. This led to many more paths being rejected than accepted for use in the database. Typically, two paths could be surveyed in one day in the field with a party of two.

Data from Norway

Data from Norway (K. Lied, personal communication 1999; Lied and Bakkehøi 1980; McClung and Lied 1987) were determined from the historical record of extreme runout from local observations and from published records for time scales of the order of 100 years, but the collection of extreme runout has a variable return period. All extreme positions were visited in the field and the angles α, β, and δ were either measured in the field using a clinometer or determined and checked by high-quality, large-scale maps, i.e., 1:5000 to 1 : 10 000 scale. Positions for the top of the start zone were estimated by field inspection.

Data from the United States

Data from the United States (A. Mears, personal communication 1999; Mears 1989; McClung and Mears 1991) were determined by vegetation damage in the runout zone, as described above for Canadian data. The data were collected by a combination of field inspection, air photograph interpretation, and topographic map analysis, and all sites were visited. The angles α, β, and δ were measured using either a clinometer or high-quality maps of large scale (whichever method was more accurate). Angles reported are assumed correct to 0.5° accuracy, in accordance with instrument accuracy and U.S. map accuracy standards. Lengths are correct to within 20 m, and RR is correct to two significant figures (Mears 1989). The maps scales ranged from 1:1200 to 1 : 24 000 with contour intervals ranging from 2 to 40 feet. The contour intervals conform to map accuracy standards in the United States which specify the margin of error statistically. Again variable return periods are expected for the data set varying between 50 and 300 years, with most exceeding 100 years.

Zoning applications for the statistical methods

Applying the empirical methods in zoning applications requires a prediction of the nonexceedance probability (the probability that avalanche runout does not exceed a given position) as a function of position along the incline into the runout zone. That is, not just the runout position is predicted, but the probability a given location will not be reached by avalanches is sought. McClung and Lied (1987) explained this concept. Given a set of extreme avalanche runouts for a mountain range, representing the historical record of extreme runout for different avalanche paths in the range, probability plots can be generated for a given avalanche path where runout is unknown with the assumption that the runout for the path obeys the probability distribution generated from the historical record at other sites. Below I give equations to calculate runout as a function of the nonexceedance probability for both the α, β regression approach and the Gumbel distribution applied to RR.

Probabilistic prediction of α and runout distance for the regression method

To apply the regression method to estimate α probabilistically, it is necessary to make an assumption about the probability density function of residuals for the regression equation. In most regression problems it is assumed that the residuals follow a Gaussian distribution, and I make this assumption here. I show later that this is a reasonable assumption for data from some of the mountain ranges, but not all. It is a valid assumption for the data sets I have used for model comparison in this paper. Thus, given a value of β for an avalanche path, α decreases as the nonexceedance probability increases with distance into the runout zone. The nonexceedance probability, *P*, at a given location then represents a position such that *P*% of α values (corresponding to runout positions) in the data set will not exceed it. The expression for α as a function of *P* is given as a confidence interval for a single response (Walpole and Myers 1978, p. 294):

[3] $$\alpha_P = C_0 + C_1\beta - z_P S\{1 + 1/N + [(\beta - \bar{\beta})^2/(S_{\beta\beta})]\}^{1/2}$$

where α_P is the value of α for a given value of *P*, *z* is the z-statistic (representing standard deviations from the mean for a standard Gaussian distribution), *N* is the number of avalanche paths, $\bar{\beta}$ is the mean value of β, *S* is the standard error of the regression equation, and $S_{\beta\beta}$ is defined as

[4] $$S_{\beta\beta} = \sum_{i=1}^{N} (\beta_i - \bar{\beta})^2$$

Note that if *P* = 0.5, eq. [3] reduces to eq. [1]; half the avalanche runouts in the data set are expected to exceed the predicted α value at *P* = 0.5. Typical values of *P* in land-use assessment applications are expected to exceed *P* = 0.9 (10% of avalanche runouts in the data set exceed the chosen position). In most cases, the second and third terms in the curly brackets are much less than 1 and eq. [3] simplifies to

[5] $$\alpha_P = C_0 + C_1\beta - z_P S$$

Equation [5] contains the prediction that, given a value of β, values of α decline according to a Gaussian distribution (or a t-distribution for data sets with fewer than 30 points) as *P* increases.

Walpole and Myers (1978) present equations to extend the prediction eq. [3] to multiple-regression problems. In practice, the value of β is found in the field, a value of *P* is specified that allows α_P to be calculated, and the runout position can be found.

In applications, the runout distance for a given nonexceedance probability is sought. Therefore, an expression for Δ*x* is needed. From eq. [6], with the geometry specified in Fig. 1, the runout distance (Δ*x*) scaled with X_β, the horizontal reach to the β point, is represented by

[6] $$\frac{\Delta x_P}{X_\beta} = \frac{\tan\beta - \tan\alpha_P}{\tan\alpha_P - \tan\delta}$$

where Δx_P is the runout distance at a given value of *P*; the quantity sought in applications. Given both β and α_p, δ is determined (Fig. 1) at a site and Δx_P may be calculated. In eq. [6], δ is not subscripted or determined statistically, it is determined analytically. This is in direct contrast to the RR–Gumbel model for which the RR is calculated from eq. [2] with random variations of α, β, and δ taken into account before *P* is estimated, as shown below.

Probabilistic prediction of runout ratio, RR

Based on data from more than 600 different extreme runout measurements from eight different mountain ranges, the RR obeys a Gumbel distribution (McClung 2000). McClung and Mears (1991), Nixon and McClung (1993), and Jóhannesson (1998) found good fits to the Gumbel distribution (see Figs. 3*b* and 4*b* for examples of the fits), particularly at exceedance probabilities greater than 0.5 representing positions where zoning applications are normally considered. The equation representing the runout ratio at a nonexceedance probability *P* is

[7] $$\mathrm{RR}_P = u + b[-\ln(-\ln(P)]$$

where *u* is the location parameter, and *b* is the scale parameter. The constants *u* and *b* may be determined by a number of different methods (e.g., Watt et al. 1989). In this paper, I have determined *u* and *b* by a least squares fit to eq. [7] by defining Hazen plotting positions in terms of the nonexceedance probability as $P_i = (i - 0.5)/N$, where *i* is the rank of the RR for avalanche path *i*. In all cases investigated thus far (McClung and Mears 1991; Jóhannesson 1998) the fits to the data on the tail of the Gumbel distribution are excellent ($R^2 > 0.94$, where *R* is the correlation coefficient). The mean of the distribution is $u + \gamma b$, where γ is Euler's constant (0.57721…) and the standard deviation is $(\pi/\sqrt{6})b$. I have given physical interpretations for *u* and *b* in terms of expected avalanche frequency and avalanche terrain steepness from one mountain range to another in a separate paper (McClung 2000). Calculations of Gumbel parameters from different mountain ranges (Mears 1989; McClung and Mears 1991) show that it is generally not valid to use Gumbel parameters determined from one mountain range to estimate runout in another.

In applications, definition of the β point gives a value of X_β for an individual path and the runout distance not exceeded with probability *P*:

[8] $$\Delta x_P = \{u + b[-\ln(-\ln(P))]\}\ X_\beta$$

which may be compared with eqs. [5] and [6] for the regression method.

Differences in empirical runout models

Equations [5] and [6] represent the regression model and eq. [8] represents the RR–Gumbel model. Three differences are apparent:

(1) For the regression method, the values of P are related to extreme runout positions in eq. [4], with the implicit assumption that the residuals obey a Gaussian distribution or another distribution. For the RR–Gumbel model, extreme runout positions are related to P according to a Gumbel distribution as in eq. [8]. These differences are reflected in runout distance calculations, as shown below.

(2) The runout distance for a given value of P depends explicitly on δ for the regression method (from eq. [6]) and implicitly on δ for the RR–Gumbel method (eqs. [8] and [2]).

(3) The runout ratio (or runout distance) for the Gumbel method is calculated from measurements of all three angles: α, β, and δ thus taking into account random variations of α, β, and δ in combination.

Calculation of Δx for the regression method takes into account random variations of α in combination with β from measurements through eq. [1], but δ is introduced analytically from eq. [6] to arrive at Δx. Thus, random variations of δ in combination with α and β are not accounted for in the regression method. In Appendix 1, I show with a multiple-regression approach that, in general, when δ is significant in a multiple regression with β to predict α, the data imply that α increases slightly (i.e., runout decreases) with increasing δ; a prediction opposite in sign and with much weaker dependence on δ than displayed by eq. [6] (shown in the next section).

Dependence of runout distance on δ as a function of nonexceedance probability

The character of the dependence of runout distance on δ is crucial for comparing the regression method with the RR–Gumbel model. Two aspects of the problem are considered here: comparison of runout predictions for different nonexceedance probabilities, and comparison of exceedance probabilities returned by different values of runout distance. In this section I consider the first of these: comparison of runout distances for different nonexceedance probabilities. In the next section I consider comparison of exceedance probability for different runout distances.

The question addressed here is: Given values of the nonexceedance probability, how do the runout distance predictions vary with δ? For this example I consider calculations with data from the Canadian Rockies with the regression equation

$$[9] \qquad \alpha = 0.93\,\beta \qquad S = 1.75 \qquad R^2 = 0.75$$

For the calculations I assume the residuals obey a Gaussian distribution (shown below in Fig. 4*a*). The Gumbel equation is

$$[10] \qquad \mathrm{RR}_p = 0.070 + 0.076[-\ln(-\ln P)] \qquad S = 0.021 \qquad R^2 = 0.96$$

In eqs. [9] and [10], S and R represent the standard error and the correlation coefficient for the least squares fits to ranked values of RR paired with values of [-ln(-ln(P)] and the α, β regression equations. For comparison of δ dependence I take $\beta = 30°$ (mean of the data) and δ in the range of 0° to 10° for three nonexceedance probabilities: 0.50, 0.90, and 0.99 with $X_\beta = 1000$ m. The results are shown in Table 1 with an equation analogous to eq. [4] used to calculate α_P using eq. [9]. The results in Table 1 show, in general, that the regression method predicts strong dependence on δ at any value of P, whereas the Gumbel method returns predictions that are independent of δ but comparable to values around $\delta = 5°$ (near the mean value for the data set).

The strong dependence of Δx on runout zone slope (δ) using the regression method is not supported by the data in this paper. For the runout distance (horizontal reach from the β point, Δx), the Spearman rank correlation coefficient (Δx with δ) is –0.25 for the Canadian Rockies.

This result is close to the correlation of RR with δ: –0.33 (shown in a later section). Therefore it seems that the runout distance has weak negative correlation with δ, based on the data, implying runout distances are nearly independent of δ increases. Similar conclusions follow from the multiple-regression relation of α with β and δ in Table A1 (see Appendix 1). Figure 2 shows a scatter plot of Δx versus δ for data from the Canadian Rockies. This figure illustrates the weak relationship between Δx and δ. Plots from other mountain ranges show results similar to Fig. 2.

The results in Table 1 indicate that, at the same nonexceedance probability, longer runout implies a more conservative prediction. For example, if $P = 0.90$, then it is predicted that 90% of avalanche runouts in the data set will not exceed a given position and 10% will exceed the position. Therefore, more slope distance will be required to reach an acceptable level of risk if predicted runout is longer for a given nonexceedance probability, implying more conservative land-use planning. It is important in zoning applications not to be too conservative because valuable land can be excluded from occupancy, but also it is important to be conservative enough that excessive risk to life and property is not incurred.

From Table 1, the most conservative method depends on both P and δ. For $\delta = 0°$, the Gumbel method (eq. [10]) is more conservative, and for $\delta = 10°$, the regression method is more conservative at any value of P. The strong dependence of runout distance on δ determined by eq. [8] dominates in Table 1, and it cannot be stated categorically that either eq. [9] or [10] provides the more conservative approach: the answer depends on both P and δ.

The results in Table 1 also show that the two prediction methods diverge at high values of nonexceedance probability (i.e., at long runout distance). The differences in Table 1 are due to the dependence on δ described above and the character of the distributions used: Gaussian distribution applied to α, and Gumbel distribution applied to RR.

Prediction of exceedance probability as a function of runout distance and δ

Next I consider a related question: Given an avalanche path, what is the probability that various positions in the runout zone are exceeded by avalanches? This is the key question in zoning applications.

In Part I of this section, I consider calculations based on data from the Canadian Rockies (eqs. [9] and [10]) and, in

Fig. 2. Horizontal reach from the β point (Δ*x*) versus δ for data from the Canadian Rockies. The scatter plot indicates a very weak relationship between Δ*x* and δ.

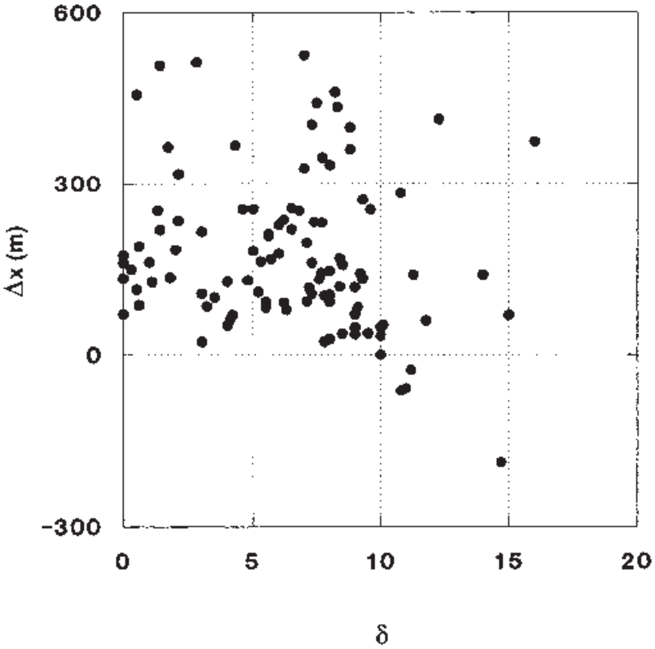


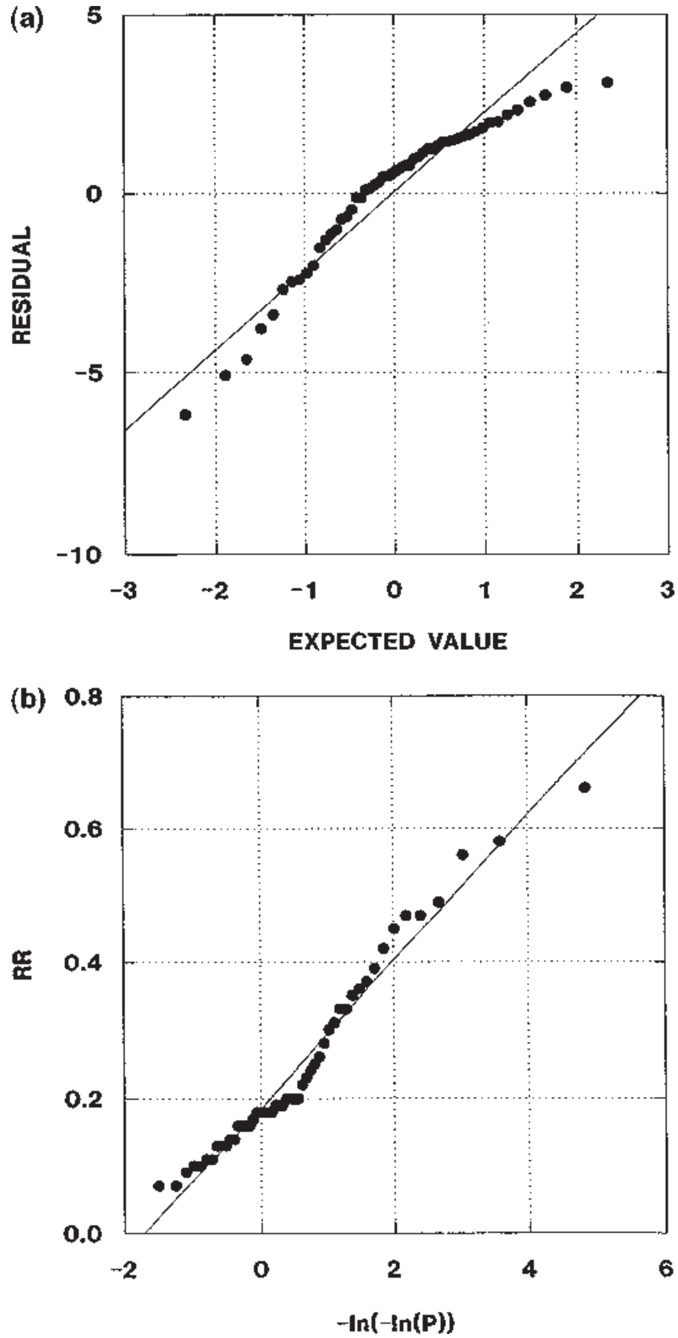
Table 1. Runout distances in metres calculated from eq. [8], regression method, and eq. [9], Gumbel method, as a function of *P* (nonexceedance probability) and the slope steepness in the runout zone, δ.

		Runout distance (m)		
P	Model	δ = 0°	δ = 5°	δ = 10°
0.50	Regression	90	108	135
0.50	Gumbel	98	98	98
0.90	Regression	202	247	319
0.90	Gumbel	241	241	241
0.99	Regression	307	382	511
0.99	Gumbel	420	420	420

Table 2. Exceedance probability (1 – *P*) versus runout distance (m) and runout slope (δ) for data from the Canadian Rockies.

Runout distance (m)	δ (°)	1 – *P* (regression model)	1 – *P* (Gumbel model)
100	0	1:2.2	1:2.0
100	5	1:1.9	1:2.0
100	10	1:1.6	1:2.0
200	0	1:9.6	1:6.0
200	5	1:5.2	1:6.0
200	10	1:3.2	1:6.0
300	0	1:84	1:21
300	5	1:23	1:21
300	10	1:8.2	1:21
400	0	1:1170	1:77
400	5	1:137	1:77
400	10	1:25	1:77
500	0	1 : 22 000	1:287
500	5	1:996	1:287
500	10	1:86	1:287

Fig. 3. (*a*) Residuals for regression α = C_1 β for Coastal Alaska. The data would fit the line if they followed a Gaussian distribution. The abscissa is in standard deviations from the mean. (*b*) Runout ratio for Coastal Alaska fitted to a Gumbel distribution; *P* is nonexceedance probability; R^2 = 0.97, *S* = 0.024.



Part II I introduce a case history from Bleie, Norway, reported by Lied et al. (1998) and examined by McClung (2000), with an estimated return period approximately 500–1000 years as a related example.

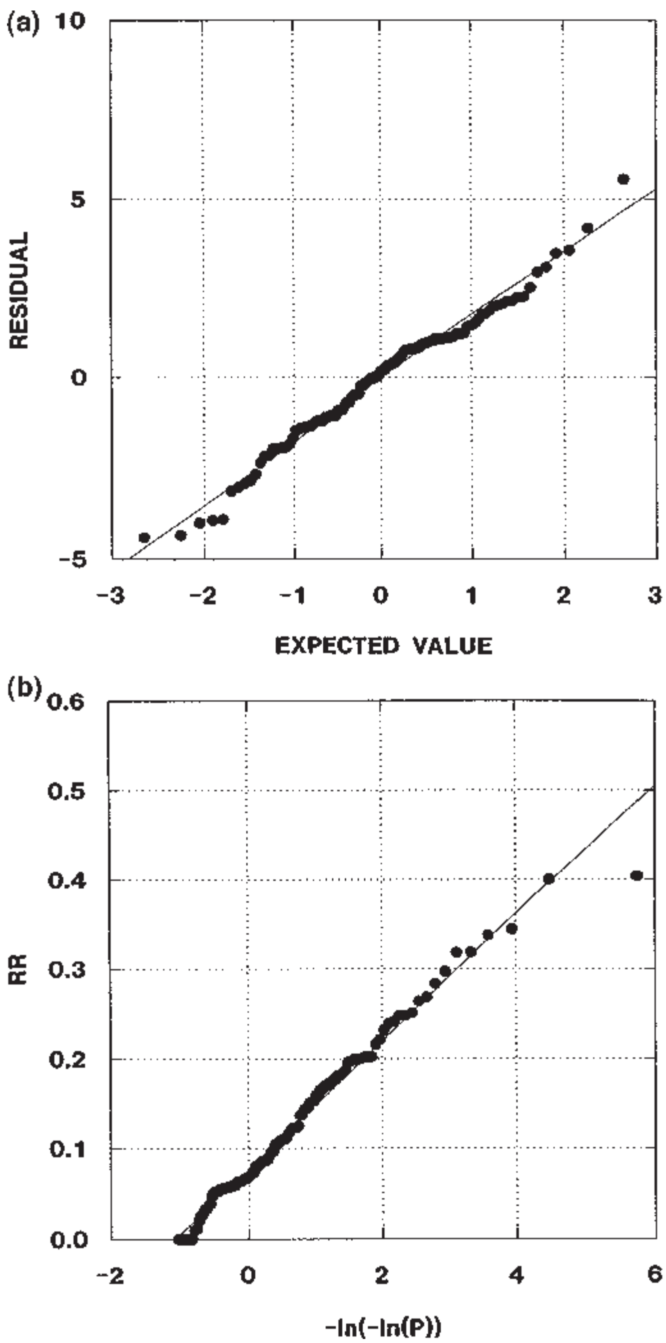
Part I: Exceedance probability as a function of δ and runout distance

The exceedance probability is related to the nonexceedance probability by 1 – *P*, and it gives a spatial esti-

Table 3. Rank correlation of RR with β and δ and significance, *p*.

Mountain Range	β correlation	*N*	*p*	δ correlation	*N*	*p*
Norway	–0.014	127	0.157	–0.154	127	0.042
Colorado	0.192	130	0.015	0.103	130	0.07
Canadian Rockies	–0.058	125	0.65	–0.330	125	< 0.001
Coastal Alaska	0.131	52	0.25	–0.266	52	0.023
Sierra Nevada	0.188	90	0.0088	0.226	90	0.017
British Columbia Coast Mountains	–0.146	30	0.216	–0.151	30	0.209

Fig. 4. (*a*) Residuals for regression $\alpha = C_1 \beta$ for the Canadian Rockies compared to a Gaussian distribution: the line. The abscissa is in standard deviations from the mean.(*b*) Runout ratio for the Canadian Rockies fitted to a Gumbel distribution for RR ≥ 0. *P* is the nonexceedance probability; R^2 = 0.99, *S* = 0.011.



mate of estimated return of avalanches at a given location. For example, if the nonexceedance probability is 0.99 at a location (runout distance), the exceedance probability is 0.01, and physically, it means that 1:100 avalanche runouts in the data set are expected to reach or exceed that location.

First, consider calculations of the exceedance probability as a function of runout distance (100 to 500 m) and δ as a companion to Table 1 with β = 30° and X_β = 1000 m. The results are presented in Table 2 with the exceedance probability presented in reciprocal form, i.e., 1:100 means 0.01 or the 1 in 100 runout. The calculations in Table 2 represent nonexceedance probabilities in the approximate range 0.5 to 0.99 similar to Table 1.

In Table 2, high values of exceedance probability represent more conservative estimates for land-use applications. For example, at 500 m for the regression method, the chance of avalanche runout exceeding the position for δ = 0° is very small (1 : 22 000); whereas for 10° the probability is much higher (1:86). If such a model were used, more conservative measures should be used for steep slopes because the probability of exceeding the position is much higher. However, in Part II of this section, I provide an example from Bleie, Norway, which indicates that the regression model implies predictions that are much too conservative for steep runout slopes far into the runout zone as in the last line of Table 2. Table 2 shows the same trend as Table 1: strong dependence on the value of δ and great differences at low exceedance probabilities. The point at 500 m is nearly 4 standard deviations from the mean of the Gumbel model, implying a very long running position with respect to the data set.

Due to the link between the nonexceedance probability and a Gaussian distribution combined through the regression method, the 500 m position in Table 2 is about 2.3 standard errors from the regression line if δ = 10°, making it the 1:86 avalanche runout compared to the data set but the 1 : 22 000 runout if δ = 0°. The differences are truly dramatic at long runout positions. As with the results in Table 1, the differences in predictions for the two methods are linked to a different dependence on δ, the differences in the two distributions used to estimate the probabilities and the calculation methods used to determine the runout positions.

The highest runout ratio from the Canadian Rockies had a runout distance of 512 m with α = 20.5°, β = 27°, and δ = 3°, with RR = 0.40. The Gumbel model predicts this is the 1:82 runout avalanche in the data set (a long running example), whereas the α, δ model predicts it is 1:179 for the data set. These results are comparable to Table 2 for which an equivalent runout ratio is found at a runout distance of 400 m. In this case, the Gumbel model produces the more conservative estimate consistent with predictions at low values of δ, as in Table 2. However, observations and human experience for

Table 4. Rank correlation of RR with AC, AS, and RW (Canadian Rockies) with significance, *p*.

Variable	Rank correlation	*N*	*p*
AC	0.244	123	0.0035
AS	0.337	49	0.0097
RW	0.277	118	0.0007

this Canadian example are not available to differentiate between the two predictions. Below I introduce a very important example from Bleie, Norway which does contain enough human experience to differentiate between the predictions of the two models.

I developed a model (McClung 2000) that combines spatial exceedance probability with temporal Poisson arrival rate upslope of the runout zone for predictions of return period as a function in the runout zone. Using exceedance probabilities from the regression method, the return period model applied to results in Table 2 would predict approximately that if the expected arrival rate was one avalanche per year at the β point: the return period would vary from approximately 86 years to 22 000 years at a runout distance of 500 m as δ varies from 10° to 0°. This compares with 287 years for the Gumbel model. These results show the regression method's enormous sensitivity on δ and the potential implications for land-use planning. Extrapolation to exceedance probabilities as low as 1 : 22 000, as implied by the regression method, is not justified based on the data set of 127 runouts for the Canadian Rockies. Such an extrapolation would be mathematically analogous to predicting a 20 000 year flood from 100 years of peak discharge stream records.

Part II: Example from Bleie, Norway, with δ = 15°

Lied et al. (1998) provided an extremely important case history from Bleie, Norway, which illustrates the strong dependence on δ implied by the regression method. This carefully documented example shows the extremely conservative dependence of the regression method model for steep runout zones and long running avalanches similar to Table 2.

According to Lied et al. (1998), a large snow avalanche reached the farms at Bleie in 1994 and they estimated by climate data analysis that the return period for avalanches is 800–1000 years at that site. Written records from Bleie date to 1293 A.D., but occupancy precedes this date. Lied et al. (1998) suggest that no major avalanches have hit the farms in the last 1000 years in spite of nearly annual occurrences that reached the β point (which is 1500 m in horizontal reach from the position of the extreme 1994 event). Another event occurred in 1776 which was long running, but this event stopped 700 m uphill from the farms. According to Lied et al. (1998): "Based on the historic evidence from the Bleie farms, with its known existence for more than 1000 years without any observations of a similar avalanche, it is clear that the return period of the disastrous avalanche must be very long."

From the data provided by Lied et al. (1998), the parameters for the extreme avalanche are as follows: α = 19.9°, β = 23.25°, δ = 15°, X_β = 2100 m, and Δx = 1500 m, which yields $\Delta x/X_\beta$ = 0.71. From the Norwegian data set: α = 0.90 β with standard error 1.9°, and the Gumbel distribution is $RR_P = 0.13 + 0.09\ [-\ln(-\ln(P))]$. These equations imply exceedance probabilities of 1:3.4 (regression method) and 1:658 (Gumbel model). The differences are dramatic, and it is within human experience at Bleie to tell which is more nearly correct. The Gumbel model predicts that the event is truly unusual with low probability.

The calculated runout ratio for Bleie is within two standard errors calculated for a Gumbel distribution (Watt et al. 1989, p. 54) of the fitted least squares line which corresponds to approximately a 95% confidence band for the line if the distribution was a normal distribution. Therefore, even though this runout ratio is the largest in the Norwegian data set, according to the methods presented by Watt et al. (1989, p. 54), it falls within a confidence band that is typical for acceptability (95%). The value of the δ angle is 15°, the highest in the data set, but there are several paths with 13° and 14° in the data set so the value of δ is comparable to other paths. The regression method predicts that the position of the Bleie farms is very risky, which contradicts human experience at the site for more than 1000 years according to Lied et al. (1998). The differences in results are similar to those in Table 2.

Distribution of residuals for the regression method

A separate, but related issue is that of the fits to the data based on the assumptions for which eqs. [4] (or [5]) and [7] are based. Equation [4] is derived from ordinary least squares with the assumption that the residuals obey a Gaussian distribution. McClung and Lied (1987) and Mears (1989) have discussed difficulties with the patterns in the residuals for some data sets. I constructed probability plots of the residuals compared to a Gaussian distribution for six mountain ranges from regression equations including data from more than 500 avalanche paths from Norway, the Canadian Rockies, Coastal Alaska, Sierra Nevada, the British Columbia Coast Mountains, and Colorado. For data from the Sierra Nevada, Colorado, the British Columbia Coast Mountains, and Coastal Alaska the residuals cannot be approximated as obeying a Gaussian distribution. Probability plots for the residuals for the Canadian Rockies and Norway show that the residuals for these data sets fit a Gaussian distribution well. Figure 3*a* shows the distribution of residuals compared to a Gaussian distribution for data from Coastal Alaska. The reason for the poor fits to the residuals from the regression equations for some mountain ranges is that sometimes α and β cannot be approximated as Gaussian variables. It might be possible to improve the fits to the residuals by transforming the variables (α and β), as suggested by McClung and Lied (1987), but this is not normally done in practice.

Figure 3*b* shows the RR data fitted to a Gumbel distribution for data from Coastal Alaska. Figures 4*a* and 4*b* show similar plots for data from the Canadian Rockies. In the latter case, the least squares residuals provide a very good fit to a Gaussian distribution, and the RR data fit a Gumbel distribution well. From all the extreme runout ratio data available, generally the RR data fit a Gumbel distribution very well, particularly for values of RR ≥ 0, as suggested by McClung and Mears (1991). However, it may or may not be the case that the residuals of the regression method provide a good fit to a Gaussian distribution. There is no assumption about the probability density function of the residuals in a least squares analysis, but when runout is estimated probabilistically an assumption about the probability density function of the residuals must be made. Mears (1989) showed similar calculations using histograms of the residuals, which deviate from a Gaussian distribution from linear regression equations. If the residuals do not obey a Gaussian distribution, eq. [3] must be modified to account for some other probability density function for the residuals.

Correlation of RR with path steepness parameters (β and δ)

A central issue of this paper is the dependence of runout on slope steepness (δ) in the runout zone. Here, I extend the analysis above to include more mountain ranges. The question of interest is: Do steeper paths tend to have larger RR or longer runout distance? Two parameters that are measures of steepness are β (steepness above the β point) and δ (steepness below the β point). Since the RR is not a Gaussian variable, I explored this question by calculating Spearman Rank correlations of the RR with β and δ for data from six different mountain ranges.

Table 3 shows, in general, that correlation of RR with path steepness is weak if either β or δ is considered. Significant positive (but weak) correlation with β is obtained for data from Colorado and the Sierra Nevada (the two ranges with the most gentle terrain on average), but for the other mountain ranges there is no significant correlation. Jóhannesson (1998) made a similar conclusion. For δ, mixed results appear, with significant positive correlation for only the Sierra Nevada and significant negative correlation for Norway, the Canadian Rockies and Coastal Alaska. Overall the correlation coefficients are small and the dependence is weak for both β and δ. Similar results are obtained for correlations of Δx with β and δ.

Correlation of RR with start zone area and runout zone width for data from Canada

In this section I consider starting zone area and runout zone width and their relation to runout distance measurements. Field experience indicates that runout distances should increase with the size of the starting zone. The analysis below shows that this is also implied for the runout ratio. For data from the Canadian Rockies information has been compiled on area of catchment (AC), area of the start zone (AS), and average runout zone width (RW). The areas of the catchment and the start zone were estimated by a combination of field observations and maps, in a method similar to other parameters defined from maps (see previous section on *Data from Canada*). The maximum runout zone width was directly measured in the field. The catchment area (AC) was defined as the maximum possible starting zone area from field observations at the sites. The start zone area (AS) was defined similarly by field observations and experience as the most likely area for the start of large avalanches. In some cases, AC and AS coincided, but in general, AC was greater than or equal to AS. There is definitely subjectivity involved in determining AC and AS, but since the data here are used only to illustrate rough conclusions with rank correlations, I feel there is value in rank correlation of RR with these parameters given the results in Table 4.

The results in Table 4 show that the RR correlates with all three parameters in a significant manner, and the interpretation may be quite simple: larger RR implies longer runout for the same X_β, which may imply potentially larger avalanches that run farther. Data from the other mountain ranges are not readily available to determine how general these results are. The results in Table 4 show that, even though the correlations are significant, the values of the correlation coefficients are quite small with only about 5–10% of the variance explained. Bovis and Mears (1976) showed with a similar analysis that starting zone area correlates significantly with runout for data from Colorado. The RR explicitly contains length information since the runout distance (Δx) correlates with length (X_β), and it would be possible to develop an equivalent model that scales Δx with H_β: height from start to the β point with results similar to those shown in Table 3.

Limitations of empirical models

The advantage of the empirical methods is simple probabilistic estimation of runout, which is needed to estimate return periods (McClung 2000). The results are predicted from the historical record in a mountain range with errors being quantified in standard statistical terms.

However, the empirical statistical models for estimating runout as discussed in this paper are subject to a host of limitations that must be recognized before applying them. Here I discuss a few that have been determined from practical experience.

More than one β point

Sometimes low-angled benches are present on avalanche paths for which the slope angle declines to 10° or below, then steepens past 10° for a section, and later declines below 10° into the runout zone. Such a profile may appear to have more than one β point making application of the models difficult. If the first bench occurs high on the path it may be possible to ignore it, based on experience, and the second may be chosen if it matches the physical expectation that it is near the beginning of the runout zone.

Path confinement

Sometimes paths are highly confined in the runout zone resulting in unusually long running distances on slopes at or below 10°. This effect will not be modelled well using data that generally do not contain such three-dimensional effects in the runout zone. For these cases, experience and three-dimensional avalanche dynamics modelling will probably be required to achieve reasonable results.

Path steepness in the runout zone

Neither of the empirical models in this paper can be relied on to give very accurate estimates of runout for steep runout zones. The results in this paper show that the least squares regression model gives unrealistic results for steep runout zones and the RR–Gumbel model does not display dependence on runout zone steepness. Troublesome paths include those which steepen after long stretches below 10°. A reasonable approach may be to estimate the probability of reaching a steep section and then apply an avalanche dynamics runout model to the steep section to estimate runout.

Continuously steep: no β point in runout zone

Some avalanche paths are continuously steep in the runout zone, and hence definition of a β point is unrealistic. Since the runout ratio may take positive or negative values (see Fig. 2), it may still be possible to employ the RR–Gumbel model. However, there are cases for which such use of the model may conflict with experience. The disaster at Valzur, Austria in February 1999 (eight people killed; several houses destroyed) may be an example. For Valzur, the runout zone is continuously steep with an average slope of about 15°, and this major avalanche stopped far above the β point (creek at valley bottom). The RR–Gumbel model may predict a very high probability of exceedance for this case, which might conflict with avalanche observations at the site.

Run-up: paths with runout zones sloping upward

In some cases the runout zone slopes upward, and in these cases it is not appropriate to use the empirical models since they are developed from data for downward sloping paths. In this case, an appropriate run-up dynamics model is recommended (e.g., McClung and Mears 1995).

Climate and terrain factors

Results of the analysis by McClung (2000) suggest that Gumbel parameters for each mountain range include terrain and snow climate effects. Examples may arise for which selected avalanche paths have terrain or climate effects that do not match those appropriate for the remainder of data from a mountain range.

Scale effects and avalanche frequency controls

The data utilized in this paper are all from large avalanche paths with vertical drops of at least 300 m with return periods between 50 and 300 years. McKittrick and Brown (1993) introduced a small data set (24 paths) from smaller paths (mean vertical drop 250 m), but they had no range of return periods associated with the data. They suggest that a different choice of β point is appropriate (slope angle 18°). It is possible that data from smaller scale paths may result in changes to the basic empirical models, in which case, an analysis similar to the one in this paper may have to be done as the conclusions may change regarding empirical modelling.

Smith and McClung (1997) performed an empirical analysis of high-frequency avalanche paths (less than 15 year return period, mean vertical drop 500 m). According to the proposal in McClung (2000) that the mean of the Gumbel distribution for a mountain range is an index of runout, the 46 high-frequency, steep paths (mean of α = 33°) of the Columbia mountains have a Gumbel mean of 0.06, which is lower than the other 8 mountain ranges reported in McClung (2000). This analysis suggests that steep, high-frequency paths have characteristics that differ from paths of comparable scale with longer return periods, and conclusions about effects of snow climate on runout for data represented by long return periods (McClung 2000) will be altered.

Discussion

For land-use applications subject to snow avalanche hazards, we wish to know the spatial probability of exceedance as a function of distance in the runout zone. The statistical models discussed in this paper provide such estimates based on terrain data collected for extreme runout specific to the mountain range in question. However, such models should not be used alone: field experience is essential in avalanche zoning, and avalanche dynamics models are applied extensively to estimate runout, often in combination with empirical models such as those discussed in this paper. Salm (1997) states: "(avalanche) dynamics models not taking probabilities into account would be worthless for hazard mapping" and he thereby reinforces the place of probabilistic models in practical applications. Furthermore, the limitations of the models described in this paper, and perhaps others, must be kept in mind.

The differences in prediction between the RR–Gumbel model and the regression method are linked to three main effects. These are as follows:

(1) The regression method predicts strong explicit dependence of runout distance and exceedance probability on the slope in the runout zone, and the RR–Gumbel model displays no explicit dependence on δ in the runout zone.

(2) Exceedance probabilities are calculated by fitting extreme runout ratios to a Gumbel distribution for the RR–Gumbel method. Exceedance probabilities for the regression method are calculated from a probability density function of residuals, often assumed to be Gaussian, from a least squares fit with α as the response variable and β as the predictor variable.

(3) Runout ratios (and hence runout distances and nonexceedance probabilities) are calculated from random variations of α, β, and δ in combination for the RR–Gumbel method: each value of RR is calculated taking all three (α, β, δ) into account.

For the regression method, values of α are calculated from random variations in β by the least squares method (excluding δ dependence), and values of P are calculated without explicit dependence on δ (e.g., eqs. [3] and [4]). The result is that the runout distance, Δx_P, is connected to δ analytically through eq. [5]. The differences in how δ is introduced explain the differences in the δ dependence on runout distance in Tables 1 and 2: no explicit dependence for the RR–Gumbel model and very strong dependence on δ for the least squares regression model.

The calculations in Tables 1 and 2, Appendix 1, and the example calculations in this paper indicate that the strong dependence of runout distance on δ exhibited by the least squares regression approach is not supported by the extreme runout data. At first glance, it may appear that the predictions of the regression model are more realistic since runout distance is predicted to increase as δ increases in the runout zone (Jóhannesson 1998). Calculations from other mountain ranges show the same dependence as shown in Fig. 2, i.e., very weak or little dependence on δ. Therefore, the data on extreme runout imply that the RR–Gumbel model provides a more realistic match.

With "everything else being equal", avalanches must run farther if the slope is steeper in the runout zone. However, the statistical data in this paper show that the dependence of runout distance on δ is weak. One important reason the physical effect of farther runout distance for steeper slopes is not shown in the data is that the constraint "everything else being equal" is violated by the data sets. The avalanche paths in the data sets should have runout zones with varying terrain roughness, degree of confinement, three dimensional terrain, different avalanche masses, different frictional character of the sliding surface and other properties when the extreme avalanches fell. Therefore, the deterministic expectation of farther running distance for steeper slope in the runout zone is not exhibited by the data. This deterministic expectation appears to be contained in the regression – least squares model but not in the Gumbel model (Jóhannesson 1998) although the analysis in this paper shows that this effect exhibited by the regression – least squares model is not exhibited by the data. Thus, even though the dependence on δ exhibited by runout distances for the regression – least squares model (increase in runout distance with increase in δ) is physically appealing (Jóhannesson 1998), it is unrealistic and unsupported by data. Such dependence should not be relied upon in applications to give reasonable results.

The example from Bleie discussed here and in McClung (2000) illustrates these points clearly. Observations at Bleie have been noted over a long enough time period to differentiate between the two models: the regression method implies that the location is risky in a probabilistic sense and the Gumbel model implies it is not very risky. Human experience agrees with the latter. The example at Bleie is important because of the long record of human occupancy, and also because δ = 15°; the long runout distance and steep runout zone slope combine to give an example for which the models differ most. With annual occurrences at the β point (Lied et al. 1998), my return period model (McClung 2000) implies a return period of about 3 years at Bleie for the regression method and about 700 years based on the Gumbel model. Thus, huge differences in the model predictions are implied for land-use planning. This example echoes the results in Tables 1 and 2, i.e., that the regression method returns more conservative results than the Gumbel method for steep runout zones, and it indicates that avalanche records at the site conflict with the predictions of the regression method.

For land-use planning, neither the regression approach nor the Gumbel approach may be said to yield more conservative results in general. In land-use applications, the most conservative prediction at a location is the one with the highest value of exceedance probability there, implying that the position of acceptable risk is farther into the runout zone and more land is excluded from human use. Often in applications, people simply choose the most conservative prediction method but such logic cannot easily be used here. Instead, the better of these empirical models should be chosen on the basis of accuracy and consistency, and the sharply contrasting dependence on δ should be considered.

Based on Table 2, the Gumbel model returns more conservative predictions for δ = 0°, and the regression method returns more conservative predictions for δ = 10° for a selection of runout distances (nonexceedance probabilities) of interest. However, the example from Bleie, Norway provides strong motivation for the belief that the predictions of the regression method are much too conservative for steep runout zone slopes far into the runout zone. Table 2 may also indicate that predictions of the regression method are not conservative enough for δ near 0° at long runout distances, but clear examples such as Bleie are not available to prove this.

I have shown in this paper, as in Mears (1989), that an assumption that the probability density function of the residuals is a Gaussian distribution is violated for some data sets. It is important to check that assumptions implicit in a model are displayed by the data. Even though this problem might be remedied by transformation of the variables, the basic differences in predictions for the two models would remain. The calculations and examples in this paper are drawn from data sets from Norway and Canada for which the residuals from ordinary least squares do fit a Gaussian distribution well, so results in this paper are not influenced by this effect.

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Appendix 1. Random variations of δ and β in relation to α.

For the regression approach, extreme runout data show that the only significant predictor for α found so far is β, unless δ is considered. Even though δ cannot predict α, it can provide information on random variations of δ and β in relation to α. In this Appendix, I explore this question using a multiple-regression approach. With δ included the simplest regression equation becomes

[A1] $$\alpha = C_0 + C_1 \beta + C_2 \delta$$

In Table A1, I provide information calculated from 6 different mountain ranges and 555 avalanche paths according to eq. [A1]. The results in Table A1 show that for all mountain ranges (except the Sierra Nevada, where dependence on δ is not significant) the addition of δ as a predictor is significant, with the general result that α increases as δ increases (positive correlation with δ). The interpretation is that statistically (but not physically) extreme avalanches that tend to stop on steeper slopes tend to be those with shorter runout. This conflicts with what is expected physically and it does not provide any insight. However, it shows behaviour about runout that is weak when the equations in Table A1 are applied using actual values of β and δ. Such calculations also show behaviour that is opposite to that for predicting α when random variations of δ are not considered (eq. [1]). When eq. [1] is used to calculate runout, without accounting for random variations of β in combination with δ, the dependence on δ enters through eq. [5] and runout distance increases strongly with δ. Therefore, when random variations of α, β, and δ are taken into account, similar conclusions are arrived at for either the regression – least squares or the RR–Gumbel model: weak statistical dependence on δ. However, since δ cannot have predictive power for the regression – least squares model, this effect cannot be employed in applications for that model. In combination with Tables 1, 2, and 3, this Appendix illustrates that the strong positive dependence of runout distance on δ for the regression – least squares model is not supported statistically by the extreme runout data. Application of the regression model, which produces results that conflict with the data it is drawn from, should be carefully considered.

Table A1. Regression coefficients (C_0, C_1, C_2), R^2, significance (p) for eq. [A1] and standard error (S).

Range	N	C_0	p	C_1	p	C_2	p	R^2	S
Norway	127	–2.48	0.029	0.94	< 0.001	0.20	< 0.001	0.89	1.74
Colorado	130	*	*	0.77	< 0.001	0.14	0.018	0.51	2.31
Canadian Rockies	125	*	*	0.90	< 0.001	0.15	< 0.001	0.80	1.57
Coastal Alaska	52	*	*	0.81	< 0.001	0.32	< 0.001	0.67	1.84
Sierra Nevada	90	*	*	0.76	< 0.001	*	*	0.58	2.33
British Columbia Coast Mountains	31	*	*	0.87	< 0.001	0.23	0.035	0.61	2.10

* Indicates the constant is not significant : $p > 0.05$.