The encounter probability for mountain slope hazards

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Abstract: In typical risk calculations for the mountain slope hazards one wishes to calculate the encounter probability: the probability of facilities or vehicles being hit at least once when exposed for a finite time period L with events having a return period T at a location. In this note, it is assumed that the events are rare, independent, and discrete, with arrival according to a binomial (or Bernoulli) distribution or a Poisson process. The constraints on the formulations for the processes are provided and it is shown that for typical applications either assumption (binomial or Poisson process) may be used in practice almost interchangeably.

Key words: encounter probability, return period, avalanches, rock fall, debris torrents.

Résumé : Dans les calculs de risque typiques concernant les dangers associés aux pentes montagneuses, on souhaite calculer la probabilité de rencontre, c'est-à-dire la probabilité que des installations ou des véhicules soient heurtés au moins une fois lorsqu'ils sont exposés sur une période de temps finie L à des événements dont la période de retour est T pour un endroit donné. Dans la présente note, on fait l'hypothèse que les événements sont rares, indépendants et suivent une loi de distribution d'arrivée binomiale (de Bernoulli) ou un processus de Poisson. On indique les restrictions sur les formulations des processus et on montre que pour des applications typiques, les deux hypothèses (loi binomiale ou processus de Poisson) peuvent être utilisées dans la pratique de manière presque interchangeable.

Mots clés : probabilité de rencontre, période de retour, avalanches, chutes de roches, torrents de débris.

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Introduction

Mountain slope hazards include, in part, rock fall, debris torrents, and snow avalanches. In many cases (e.g., Smith and McClung 1997), it is a reasonable assumption that they can be modelled as discrete, rare, independent events such that the probability of occurrence during any time interval Δt is a small number and the probability of two or more events in any time interval is negligible.

Applications such as risk calculations may contain estimates of the encounter probability: the probability an event occurs at least once during a finite time interval L if the return period at the location is T. In British Columbia, land-use zoning restrictions are often applied by estimates of the encounter probability, for example, 10% chance of at least one occurrence in 50 years (implying a return period of 475 years). In this note, I consider the encounter probability from the dual perspective that events arrive according to a Poisson distribution or that arrival follows a binomial distribution. In particular, the focus is on the constraints on the equations derived, the physical interpretation, and return-period limits of the derivations. The results show that computationally either formulation may be used in most cases but the physical interpretations of the formulations differ.

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Binomial distribution

Assume that the probability of an event (rock fall, debris torrent, snow avalanche) during any time interval, Δt, is small (the probability of two or more events during Δt is negligible), then the total number of events during the finite time interval $L = n\Delta t$ is described by the binomial probability mass function (Benjamin and Cornell 1970, p. 224):

$$[1] \qquad P_K(k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \qquad k = 0, 1, 2, \ldots, n$$

where n and k are finite integers. In eq. [1], $P_K(k)$ represents the probability that events occur k times (k is a random number) in n independent time intervals (trials) Δt in the finite time interval L. The probability, p, of an event occurring during any independent time interval is

$$0 \le \frac{\Delta t}{T} \le 1$$

and the expected number of events during n time periods is $(n\Delta t)/T$.

From eq. [1], the encounter probability, E_p, is the probability of occurrence at least once in n trials. It is the sum of all terms except $k = 0$ to yield

$$[2] \qquad E_p = 1 - \left(1 - \frac{\Delta t}{T}\right)^n$$

Bunce et al. (1997) and Hungr and Beckie (1998) considered an application which requires eq. [2]: traffic is stopped once per year in a rock-fall zone for 0.5 h along a highway with 2.2 rock falls on average per year (T = 0.4545 years per event), then for Δt = 0.5 h (1 : 17 520 years) and n = 1 (0.5 h period), E_p = 1 : 7964 for the rock-fall zone. If cars occupy 78% of the linear distance along the highway as described by Bunce et al., then each year there is a 1 : 10 210 chance of a vehicle being hit by a rock fall.

Poisson process

Consider a Poisson process over a finite time period $L = n\Delta t$ with a return period T. In this case, the process may be thought of as many very small time intervals, Δt, such that as $n \to \infty$, $\Delta t \to 0$, but the product (total time interval) $L = n\Delta t$ is finite as assumed for the binomial distribution. For such assumptions, the Poisson probability mass function may be written (with the Poisson parameter $\mu = L/T$) as

$$[3] \qquad P_K(k) = \frac{\mu^k \exp(-\mu)}{k!} \qquad k = 0, 1, 2, \ldots, \infty$$

where k is a random number of events, and μ is the expected Poisson arrival rate during L.

From eq. [3], the sum of all terms except $k = 0$ gives the encounter probability

$$[4] \qquad E_p = 1 - \exp\left(-\frac{L}{T}\right)$$

Properly, eq. [4] may be applied for any return period $0 \le T \le \infty$. If $T \to 0$, then $E_p \to 1$, and if $T \to \infty$, $E_p \to 0$. For the example above, application to the rock-fall problem considered by Bunce et al. (1997, 1998) and Hungr and Beckie (1998), using the binomial distribution, calculation of E_p with the Poisson expression (4) is identical to that for the binomial distribution with L = 1 : 17 520 and T = 0.4545 to give E_p = 1 : 7964. LaChapelle (1966) considered numerical comparisons of equations similar to eqs. [2] and [4] and he showed very small differences over typical ranges of parameters for snow avalanche applications.

Return-period limits of the binomial distribution

As with the Poisson process, the binomial encounter probability may be applied within the limits $0 \le T \le \infty$. From eq. [2], if $T \to \infty$, then $E_p \to 0$.

For the other limit as $T \to 0$, with $L = n\Delta t$ finite,

$$[5] \qquad E_p = 1 - \left(1 - \frac{L}{nT}\right)^n$$

Now as $\Delta t \to 0$, n must approach ∞ such that the constraint $0 \le \Delta t \le T$ is followed. From eq. [2], if $T \to 0$, it is implied that $n \to \infty$, from which

$$[6] \qquad E_p = 1 - \lim_{n\to\infty}\left(1 - \frac{L}{nT}\right)^n = 1 - \exp\left(-\frac{L}{T}\right)$$

and from eq. [6], $E_p \to 1$ as $T \to 0$.

In practice, for typical return periods of interest and finite time intervals L, eqs. [2] and [4] provide estimates which differ less than data accuracy for calculating return-period estimates. Some engineers and scientists may prefer the expression given by eq. [4] for the Poisson process because it may be applied directly for any return period $0 \le T \le \infty$, whereas eq. [2] requires application of limits with Δt decreasing as T decreases in the limit as $T \to 0$ (infinitely many events). Computationally, the encounter probability for the Poisson process or the binomial distribution could be applied for finite Δt and finite n but physically E_p for the Poisson process contains the assumption $n \to \infty$ as $\Delta t \to 0$ with L finite, so such computations lack a physical basis for the Poisson process.

Summary

The encounter probability may be derived from the Poisson distribution or the binomial distribution for a finite interval of time $L = n\Delta t$ for which facilities or vehicles are exposed, and very similar results are obtained in most cases. For the Poisson distribution, L is physically divided into many time intervals (Δt) of very small duration, whereas for the binomial distribution L may consist of a finite number of time increments of duration Δt, each of which constitutes one Bernoulli "trial."

For either the binomial or Poisson encounter probability, the constraint $0 \le \Delta t \le T$ must be followed. For both distributions $E_p \to 0$ as $T \to \infty$ and $E_p \to 1$ as $T \to 0$. However, as $T \to 0$ the binomial encounter probability converges to the Poisson encounter probability first and then the limit is applied.

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