

## 24 Snow Avalanches as a Non-critical, Punctuated Equilibrium System

D.M. McClung

Department of Geography, University of British Columbia, Vancouver, B.C. V6T 1Z2  
Canada, mcclung@geog.ubc.ca, Tel: 604-822-9157, Fax: 604-822-6150

**Abstract.** The mathematical requirements for a self-organized critical system include scale invariance both with respect to the characteristic sizes of the events and the power spectrum in the frequency domain based on time arrival. Sandpile avalanches and other types of avalanches have been analyzed from the perspective of common characteristics of critical systems. However, snow avalanches have not been completely analyzed, particularly in the frequency domain. Snow avalanches constitute a natural hazard and they are of much more practical importance than other types of avalanches so far analyzed. In this chapter, I consider the mathematical criteria for scale invariance in both the size and frequency domain for snow avalanches based entirely on analysis of field measurements.

In combination, the mathematical results suggest that neither the size distribution, the time arrival nor waiting time between avalanche events conform to that of a critical system as defined for self-organized criticality or thermodynamics. If snow avalanches are to conform to a critical system in geophysics then a revision of the mathematical requirements or definition is called for. However, time series of events show that snow avalanche arrivals consist of clusters, intermittencies and bursts with rapid changes over short time intervals interrupted by periods of stasis. The data and analysis combined with field observations suggest that a system of snow avalanches paths exhibits the characteristics of a non-critical, punctuated equilibrium system.

### 1 Introduction

Bak, Tang and Wiesenfeld (1987) provided the original description of a self-organized critical system. They defined self-organized criticality (SOC) for complex systems as implying that no single characteristic event size or time scale exists. The essential meaning is that there is not just one time scale or length scale that controls the temporal or spatial evolution of such a system. According to Jensen (1998): "For a critical system, the same perturbation applied at different positions or at the same position at different times can lead to a response of any size." The original application in earth geophysical systems was applied to sandpile avalanches by a hypothetical computer model. Subsequently, it was shown that SOC does not apply to sandpile avalanches when they were observed instead of relying on computer modelling. Since the term "avalanche" is used quite often in description of SOC, it is

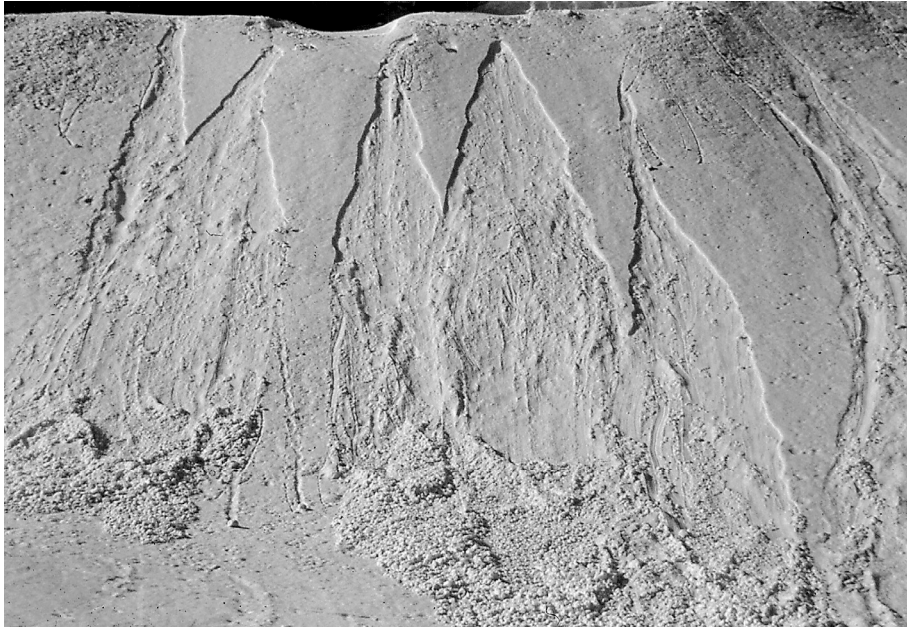
a fair question to ask about applicability to snow avalanches since they occur frequently and they have important effects on human life causing death and destruction. If SOC applies (or if it does not apply), then important implications for avalanche forecasting are implied. In common with the analysis of sandpile avalanches, field observations and measurements are used in this chapter instead of theoretical conjecture or computer models to test if SOC applies to snow avalanches.

This chapter is divided into two general parts to check the mathematical requirements for self-organized criticality to apply. The first part deals with examination of scale invariance in relation to sizes of snow avalanches and the second part deals with time arrival and waiting time to estimate the power spectrum in frequency domain. The application to snow avalanches is of some importance since snow avalanches along with earthquakes constitute two of a very few geophysical systems with a high enough frequency of events to perform analysis for both the size and frequency (or time) domain. Landslides, debris flows and rock avalanches, for example, do not have a high enough frequency of events for analysis in the time/frequency domain.

Most claims about the applicability of SOC (Bak 1996) involve checks on either scale invariance of the power spectrum in the frequency domain (power law dependence) or scale invariance with respect to event sizes (power law behaviour of some quantity which represents size). However, Bak, Tang et al. (1987) and Jensen (1998) emphasize that scale invariance in both the frequency and size domain are needed to demonstrate SOC. In this chapter, I suggest through use of field data from snow avalanches that neither of these mathematical requirements are met. However, some of the descriptive aspects of SOC appear to have merit based on field observations and measurements including large catastrophes and bursts of activity over short time scales which are characteristic of punctuated equilibrium systems.

## 2 Loose Snow Avalanches: Size Information

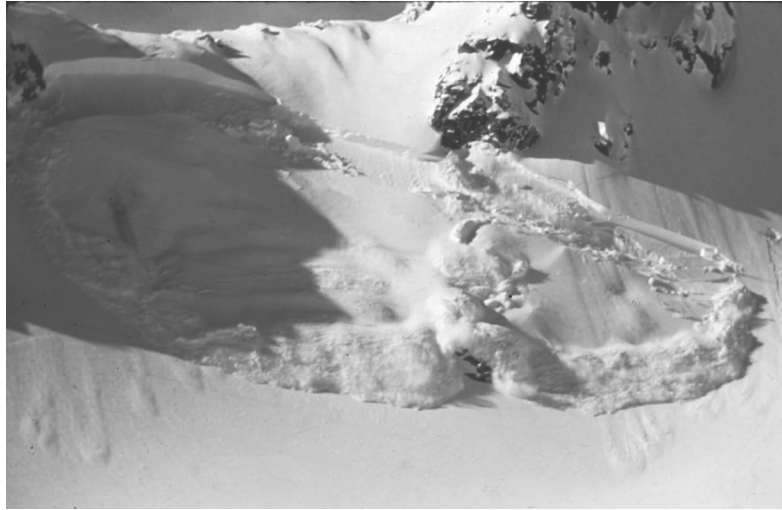
Loose snow avalanches form [see (McClung and Schaerer 2006) for details on loose snow avalanches] in cohesion-less snow at the surface and are analogous to sandpile avalanches. When they occur (see Fig. 24.1), they are generally of nearly the same size. I suggest that this is due to inertial effects. Even though they may initiate with different sizes, the final size is determined by inertial effects and they sweep out snow of similar size (volume). There are no data to show this effect in general. However, field observations suggest it is a fairly general trend and Fig. 24.1 supports the idea. Without a spectrum of sizes then, I suggest that loose snow avalanches share a characteristic with sandpile avalanches which violates SOC. Namely that final sizes are controlled by inertial effects and implicitly SOC should not apply since the final masses are nearly all the same size during times when the physical condition of the surface layers remains roughly the same.



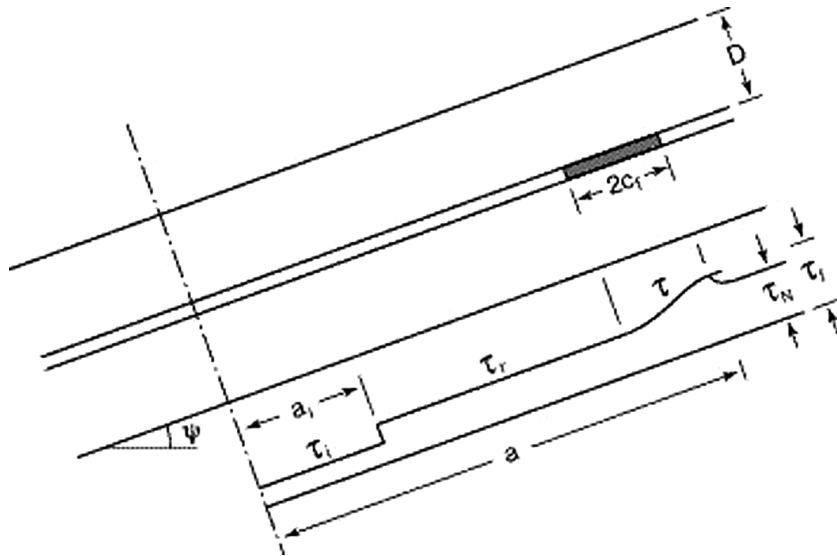
**Fig. 24.1.** Loose snow avalanches. Photo: SLF archives. This example shows they are all nearly the same size.

### **3 Scale Invariance with Respect to Multiplicative Changes in Slab Thickness**

Slab avalanches (Figure 24.2) initiate by shear fracture propagation (McClung, 1979; 2005) within a thin weak layer at a depth  $D$  (the slab thickness) below the snow surface (see Fig. 24.3). In general, they mostly release due to snow loading during storms or from loading by blowing snow (see (McClung and Schaerer 2006) for an extensive review of slab avalanche formation). The depth to the weak layer,  $D$ , is similar for most slab avalanches that release in storms in an area, with the result that there is a characteristic length scale associated with release. Thus, when they occur in an area, they are mostly of the same size. The characteristic length scale for release violates the principles of SOC, so I suggest that it is highly unlikely that SOC applies to slab avalanche release. No analysis is needed to make this claim but field observations and measurements confirm the existence of the thin weak layer and its role in the introduction of a length scale ( $D$ ) which is involved in slab release.

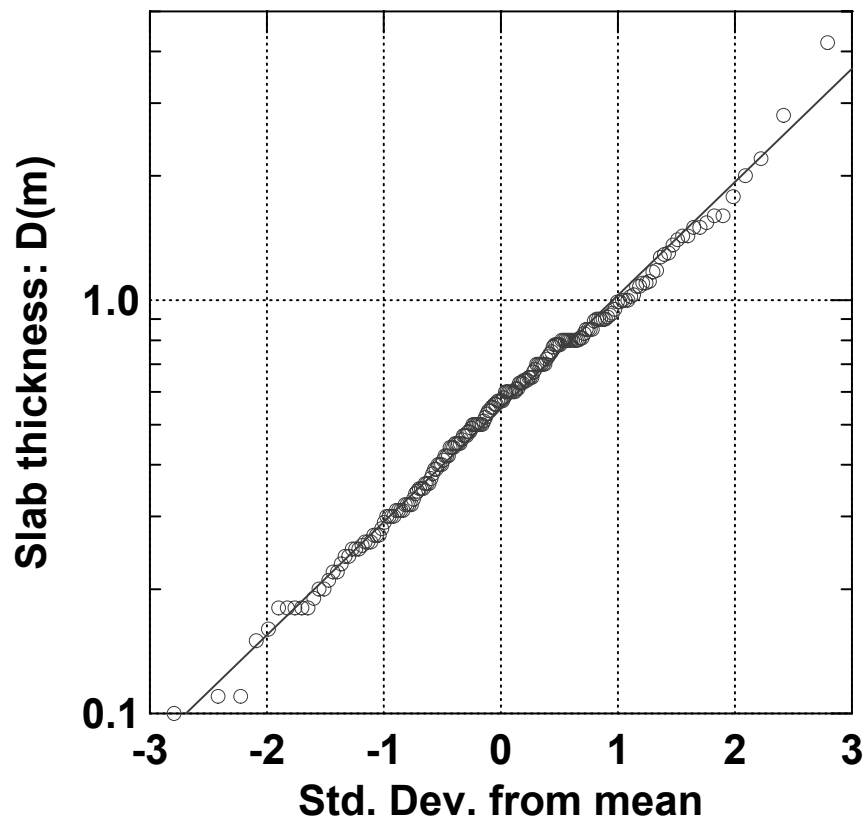


**Fig. 24.2.** Example of slab avalanche release , Whistler Bowl, British Columbia. Photo by T. Salway. The avalanche releases by shear fracture at depth  $D$  under the slab.



**Fig. 24.3.** Schematic for release of a snow slab on a slope of angle  $\psi$  after Bažant et al. [2003]. Strain-softening occurs in shear with the fracture process zone ( $2c_f$ ) of finite length in a weak layer at depth  $D$ . Failure initiates at an imperfection of length  $a_i$  with shear stress  $\tau_i$  in the weak layer. The weak layer has failure stress  $\tau_f$ , residual stress,  $\tau_r$  and  $\tau_N$  is applied nominal shear stress.

It is still of interest to analyse the distribution of sizes, particularly, the fundamental size parameter  $D$ . Figure 24.4 shows a probability plot of  $D$  for 191 slab avalanches. The data suggest that the fundamental length scale,  $D$ , follows a log-normal probability density function. The data in Figure 24.4 are from mix of avalanche triggers including: snow loading (called naturals), skier triggering and explosive loading. I have analysed (McClung 2003 a) other data sets from skier triggering and natural loading and the results are similar. In each case,  $D$  is log-normally distributed. Since the data range only from about 0.1 m to about 4 meters, it is not possible that the data follow a power law or scale invariant form (Pareto distribution) over any significant range of  $D$  (e.g several orders of magnitude). I have also made similar plots for slab length and slab width and they also display log-normal form. Thus, I conclude that the application of SOC fails with respect to size for slab avalanches for two reasons: 1. there is an implicit length scale:  $D$ , the slab



**Fig. 24.4.** Probability plot of slab avalanche thickness  $D$  (m). The plot suggests  $D$  follows a log-normal probability density function. The abscissa represents the  $z$  score in standard deviations from the mean.

thickness associated with release and this violates the fundamental assumptions of SOC; 2. The actual sizes do not display scale invariant (power law) behaviour over any significant range of the size scale  $D$  or other size characteristics such as slab length and width. I have discussed this in detail (McClung 2003 a).

#### **4 Analysis in the Time and Frequency Domain for Slab Avalanches**

Snow slab avalanches, along with earthquakes, constitute two geophysical systems with a high enough frequency of events to enable analysis for scale invariance in the frequency or time domain to check the assumptions required for a self-organized critical system. Bak et al. (1987) suggested that a self-organized critical system should display scale invariance of the power spectrum in the frequency domain with proportionality to  $1/f$  or  $1/f^\beta$  where  $\beta$  is a constant that is usually between one and two. If such dependence was proven then no characteristic time scale would be available. For avalanches, this could affect the fundamental properties of avalanche forecasting. The idea that geophysical systems might have some simple common properties is also appealing. Since the ideas about self-organized criticality started with analysis of sandpile avalanches, it is useful to check the assumptions with full scale snow avalanche events as a natural geophysical system. In addition to checking the assumptions of self-organized criticality, such an analysis should provide fundamental information about time scales, whether they are present or not, in relation to avalanche forecasting.

McClung (2003(b)) provided an empirical analysis of avalanche time arrival from Bear Pass and Kootenay Pass in British Columbia from thousands of avalanches based on 20 years of records. The analysis revealed that a time scale of about 1 day appears from the times series analysis of arrivals which does not conform to the scale invariance in time postulated for a self-organized critical system. This results of the analysis are briefly reviewed in the next section (5). The reader is referred to McClung (2003(b)) for all the details of the analysis.

In this chapter, another analysis is performed from data bases of avalanche arrivals for Bear Pass and Kootenay Pass. The analysis here is done from the perspective of an analysis of waiting time between snow avalanches to check for

$1/f$  dependence in the frequency power spectrum. The average waiting time between events is calculated for 110 different avalanche paths from 15,990 avalanches over 23 winter seasons in which approximate time of occurrence is known. The waiting time analysis is given in sections 6-9 below.

## 5 Review of Empirical Arrival Time Analysis for Bear Pass and Kootenay Pass

McClung (2003(b)) calculated time arrival for avalanche events for determination of the frequency power spectrum for Bear Pass and Kootenay Pass. Direct calculation from the time series of events collected in 12 hour intervals showed that the auto-correlation function had exponential form. Each element of the series consists of sum of the masses of all avalanches recorded within 12 hour intervals. The sizes of the avalanches according to the Canadian avalanche size classification (McClung and Schaerer 2006) were recorded by technicians and these were then converted to mass with all masses summed over 12 hour intervals to yield time series for each pass. See McClung (2003(b)) for more details.

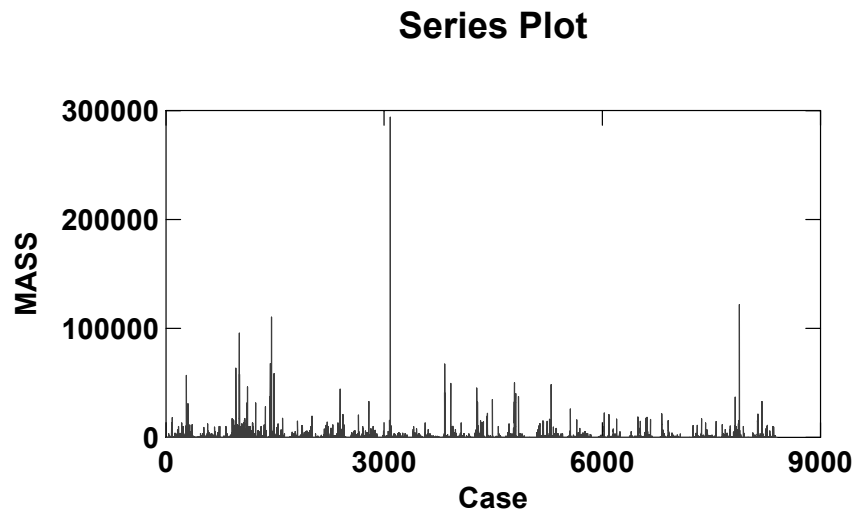
The auto-correlation function for such a time series is an empirical relation derived by correlating the time series with itself displaced by one or more lags. By definition, the auto-correlation function is 1 at zero lag since it represents the series correlated with itself. In this manner, the auto-correlation function is equivalent to an empirical histogram of the time series correlated with itself for lags 0,1,2,... where in this case one lag equals 12 hours. The empirical form derived from the time series of events (McClung 2003(b)) after the constant (d.c.) component was removed is:

$$c(t, \tau_0) = \exp(-t / \tau_0) \quad (1)$$

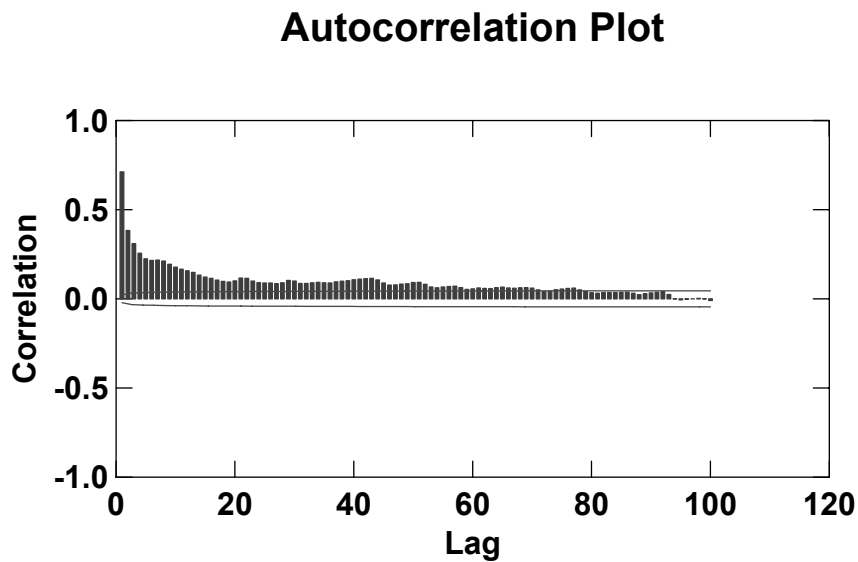
where  $\tau_0 \approx 1-2$  days (the e-folding time) for both Bear Pass and Kootenay Pass was shown by (McClung, 2003(b)). Figures 24.5 shows the time record of total avalanche mass (t) summed for all avalanche paths at Bear Pass for 20 years of records from 1981 -2001 with data stratified into 12 hour time periods where 12 hours constitutes a lag of one. Summer periods from April 30 - November 1 were removed. It shows that the time record of summed avalanche arrivals (all events are naturals without human triggering) consists of a series of clusters, intermittencies and bursts. Figure 24.6 shows the empirical auto-correlation plot obtained directly from the time series from (McClung 2003(b)). Similar plots are available for Kootenay Pass (McClung, 2003(b)). The frequency power spectrum associated with equation (1) is:

$$S(f, \tau_0) = 4 \operatorname{Re} \int_0^{\infty} c(t, \tau_0) e^{2\pi i f t} dt = 4\tau_0 / [1 + (2\pi f \tau_0)^2] \quad (2)$$

This analysis suggests a finite time scale ( $\tau_0 \approx 1-2$  days) associated with the avalanche arrivals with transition to white noise for low frequencies. Typical storm durations generating avalanches can last from a fraction of a day to as long as about 10 days in the avalanche areas considered here. These empirical results suggest that scale invariant behaviour in the frequency power spectrum for time arrival is not expected over any significant range of frequency or time. McClung (2003(b)) provides further details to generalize this result for the system modelled as a shot noise Poisson process.



**Fig. 24.5.** Time series of summed avalanche mass (tonnes) for all the avalanche paths at Bear Pass, British Columbia for data from 1981 -2000. Each case consists of a 12 hour period (one lag). Summer periods were removed to provide an approximately stationary time series. The data consist of natural events unaffected by human avalanche control.



**Fig. 24.6.** First order auto-correlation function for time arrival of events for 20 years of records from Bear Pass, from (McClung 2003(a)) for the time series of Figure 24.5. The horizontal lines represent the limits of two standard errors (or 95% confidence limits): correlations outside inside these lines are not significant. One lag equals 12 hours.



Conclusions about frequencies higher than the Nyquist frequency (once a day) cannot be made for the analysis since the effective sampling rate is twice per day (12 hour intervals or lags). McClung (2003(b)) found by non-linear fits to the auto-correlation function that the exponential decay of the auto-correlation function as in equation (1) provided a better fit than a half-Gaussian function for both Bear Pass and Kootenay Pass. I suggest the plots in Figures 24.4 and 24.5 illustrate the after-effects of random perturbations applied by storms to generate avalanche arrivals for the systems of avalanche paths at Bear Pass and Kootenay Pass with decline in correlation to white noise (constant power spectrum) over a limited interval in a time of several and perhaps up to ten  $\tau_0$ .

## 6 Waiting Time Analysis as a Poisson Process

The results above suggest that there is a time scale associated with summed avalanche arrivals when the collection of avalanche paths is viewed as a system. This is in agreement with field observations and data that almost all avalanches arrive during or within a day or so after storms. The time series analysis suggests that an avalanche area viewed as a system has short time memory and the after effects of random perturbations (storms in this case), correlation declines as  $\exp[-t/\tau_0]$  with the power spectrum as in equation (4). Clearly this does not represent scale-free invariance in the frequency domain as envisioned for SOC by Bak et al. (1987).

Another possibility for analysis in the time/frequency domain is to analyse waiting time between avalanches. Ito (1995) suggested scale invariance in the frequency power spectrum by an analysis of earthquake waiting times in California. Thus, it is of interest whether such behaviour might apply to snow avalanches. In this section, I performed the analysis for large data bases of avalanche occurrences based on 23 years of records from Bear Pass and Kootenay Pass. Below, I attempt to analyse correlations in the time/frequency domain by decomposing the signal into a linear random superposition of independent components. The components consist of individual, independent avalanche paths.

I assume (McClung 1999) that avalanches for a given avalanche path arrive as random, rare, discrete independent events such that the probability of two events arriving simultaneously is negligible. These conditions fit avalanche arrivals and they conform to the assumptions of a Poisson process. If the random variable  $T$  is the time to the first arrival, then the probability that  $T$  exceeds some value of time  $t$  is equal to the probability that no events occur in  $t$ . The probability of exceedance is:  $1 - P_i(t)$  where  $P_i(t)$  is the cumulative distribution function for avalanche path,  $i$ . For Poisson arrivals, the probability of no events in time  $t$  is:

$$e^{-\frac{t}{\tau_i}} \left( \frac{t}{\tau_i} \right)^0 \frac{1}{0!} = 1 - P_i(t) \quad t \geq 0 \quad (3)$$

Where  $\tau_i$  is the average waiting time between events for avalanche path  $i$ . From equation (3), the probability density function for path  $i$  is (Benjamin and Cornell 1970):

$$\psi_i(t) = \frac{dP_i(t)}{dt} = \frac{1}{\tau_i} \exp\left(-\frac{t}{\tau_i}\right) \quad (4)$$

Since for a Poisson process there is no memory of past events, the analysis applies for any interval of time whether or not it begins at  $t = 0$ . Thus, if time is counted from the last event ( $n$ th), then  $\exp(-t/\tau_i)$  is the exceedance probability or the probability that the time to the  $(n+1)$ th event is greater than  $t$ . It is important to distinguish between the constant  $\tau_i$  (the average waiting time between events for an individual path  $i$ ) and the empirically found  $\tau_0$  in the analysis above which refers to time arrivals for the entire avalanche areas at Bear Pass and Kootenay Pass. The latter reflects time arrivals and it is closely related to a characteristic relaxation time for the system of avalanche paths as a whole.

## 7 Description of Data for Waiting Time Analysis

The data used in the analysis consist of avalanche events recorded at two avalanche areas in British Columbia (B.C.), Canada. The times of avalanche events were recorded between November 1, 1981 and April 30, 2005 for sets of avalanche paths at Bear Pass in northwest B.C. and Kootenay Pass in southeast B.C. The data base consists of 15,990 avalanches recorded for these areas for 23 winter seasons from November 1 each year until April 30 each year. Data from all avalanche paths for these two areas were included except at Bear Pass two paths were excluded which had very high frequency of events with multiple gullies spawning small avalanches so they were not distinct paths. Also, events recorded as loose snow avalanches were excluded from the analysis so that only slab avalanches were considered.

Since the snow and avalanche season is generally from November 1 - April 30, events outside these dates were excluded. Thus, the data sets can be envisioned as having a stationary continuous flow of events over the time scale used (1981 - 2005) with the summer time periods removed as described by McClung (2003(b)) for the arrival analysis above. There are occasional events before November 1 and after April 30 but generally they are not recorded and omission of these few events should not affect the analysis given here.

Average waiting times between events were calculated for each avalanche path by determining the total number of avalanches recorded over the 23 season time period and the total time between the first and last events recorded. Avalanche paths with

only one event over the entire 23 years were excluded since it was not possible to determine a value for the time between events. The average time between events for an avalanche path was determined by calculating the time range over which events were recorded and then the number of summer periods through the time range were subtracted. This total time was then divided by :  $N_0 - 1$  where  $N_0$  is the total number of events recorded for a path. The data set included 5855 avalanches from 37 paths at Kootenay Pass and 10,135 avalanches from 73 paths at Bear Pass. Table 24.1 contains descriptive statistics for the two avalanche areas with mean and standard deviation of waiting times for the avalanche areas given.

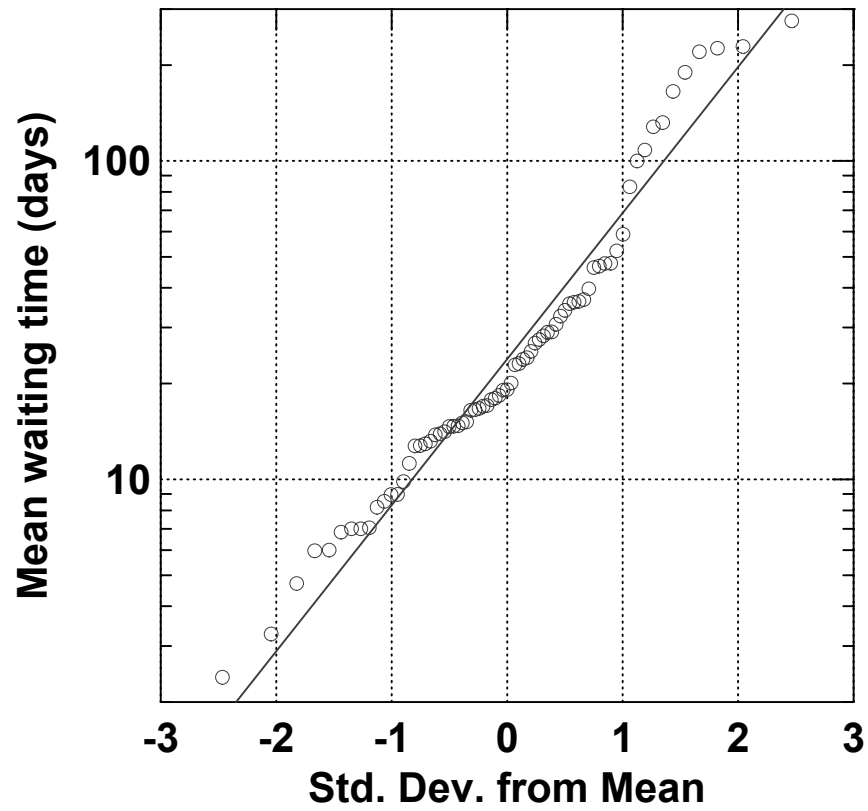
**Table 24.1.** Descriptive statistics for Bear Pass (B. Pass) and Kootenay Pass (K. Pass) with mean and standard deviations (  $\overline{\tau_i}, \sigma_{\tau_i}$  ) of waiting time given for avalanches (Aval.).

Area	No. Aval.	No. Paths	$\overline{\tau_i}$	$\sigma_{\tau_i}$
B. Pass	10135	73	43 days	59 days
K. Pass	5855	37	94 days	134 days

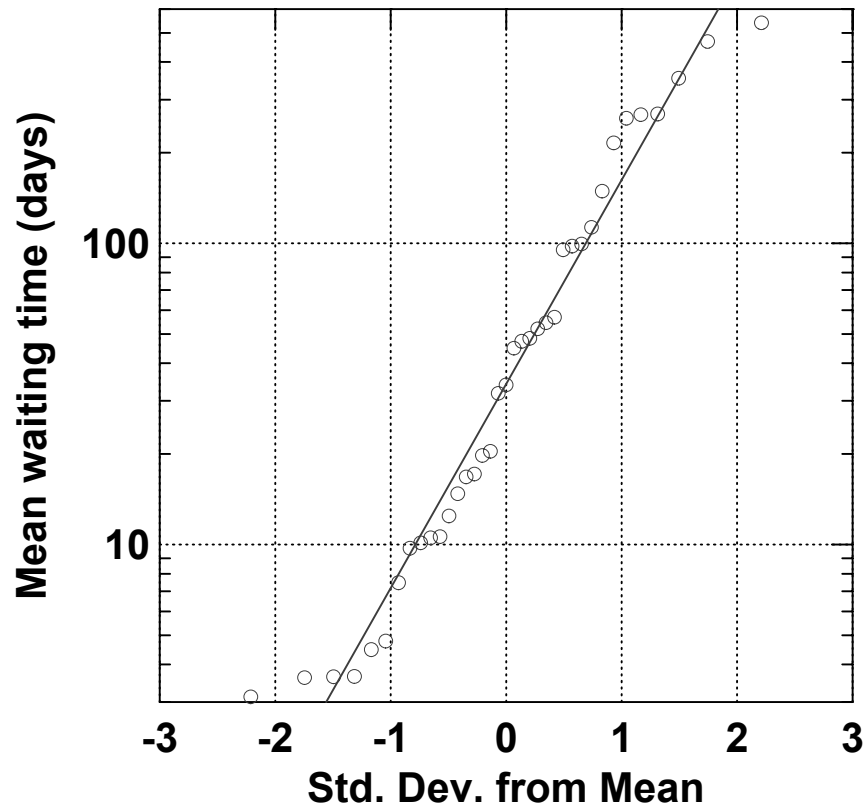
Figures 24.7 and 24.8 contain probability plots of mean waiting times for the avalanche paths. In both cases, the ordinate is logarithmic so that the plots suggest the waiting times follow log-normal probability density functions. Table 24.2 contains relevant descriptive statistics.

**Table 24.2.** Mean and variance of  $\ln(\tau_i)$  for Bear Pass (B. Pass) and Kootenay Pass (K. Pass) from 15,990 avalanches recorded over 23 winter seasons.

Area	No. Paths	$\overline{\ln(\tau_i)}$	$\sigma^2_{\ln(\tau_i)}$
B Pass	73	3.1	1.1
K Pass	37	3.5	2.3



**Fig. 24.7.** Probability plot of mean waiting time for 73 avalanche paths from Bear Pass. The plot suggests that mean waiting time has a log-normal probability density function for the avalanche area. The abscissa represents the z score in standard deviations from the mean.



**Fig. 24.8.** Probability plot of mean waiting time for 37 avalanche paths from Kootenay Pass similar to Fig. 24.7. The plot suggests that mean waiting time has a log-normal probability density function. The abscissa represents the z score in standard deviations from the mean.

## 8 Effects of Avalanche Control

The avalanche paths at Kootenay Pass (K. Pass) are heavily controlled with gunfire much more so than at Bear Pass (B. Pass). In order to assess the effects of avalanche control, the analysis above was repeated for natural occurrences only. The results are given in Tables 24.3 and 24.4 which may be compared with Tables 24.1 and 24.2.

**Table 24.3.** Descriptive statistics for natural events (Aval.) only for Bear Pass (B. Pass) and Kootenay Pass (K. Pass) with mean and standard deviations  $(\bar{\tau}_i, \sigma_{\tau_i})$  given.

Area	No. Aval.	No. Paths	$\bar{\tau}_i$	$\sigma_{\tau_i}$
B. Pass	8550	73	54 days	82 days
K. Pass	1225	35	96 days	135 days

**Table 24.4.** Mean and variance of  $\ln(\tau_i)$  for 9775 naturally occurring avalanches recorded over 23 seasons for Bear Pass (B. Pass) and Kootenay Pass (K. Pass).

Area	No. Paths	$\overline{\ln(\tau_i)}$	$\sigma^2_{\ln(\tau_i)}$
B Pass	73	3.3	1.1
K Pass	35	4.5	1.0

Comparison of the tables shows that far fewer avalanches are naturals for Kootenay Pass and the variance  $\sigma^2_{\ln(\tau_i)}$  is much lower for Kootenay Pass when only natural events are considered from Kootenay Pass. The variance is comparable to Bear Pass if explosive controlled events are eliminated. Probability plots for natural events only are very similar to Figures 24.3 and 24.4 and they imply that the collection of mean waiting times  $\tau_i$  follows a log-normal distribution for either avalanche area.

## 9 Rare Events Approximation for Waiting Time Analysis

The analysis in this paper is done from the perspective to the rare events approximation (McCormick 1981) applied to the avalanche areas for avalanche waiting times. The rare events approximation consists of formulations for the intersection of events and the union of events applied to sets of avalanche paths at Bear Pass and Kootenay Pass. The basic assumption is that occurrences are independent and highly infrequent. It is assumed that the probability of two or more events is small compared to the probability for any single event. These assumptions will apply to events on a given avalanche path. For the entire avalanche areas consisting of 73 paths (Bear Pass) and 37 paths (Kootenay Pass), I have found (see Appendix A) that the union of events for the rare events approximation holds over only a very short restricted time scale (order of a few hours) whereas the intersection

of events should be valid without restrictions since all the avalanche paths are independent. Consider an avalanche path  $A_i$  and let  $P(A_i)$  be the probability of an event on this path. For paths  $1, N$ , the union of events is then:

$$P(A_1 \cup A_2 \cup \dots A_N) \approx \sum_{i=1}^N P(A_i) \quad (5)$$

and the intersection is given by:

$$P(A_1 \cap A_2 \cap \dots A_N) = P(A_1)P(A_2) \dots P(A_N) \quad (6)$$

Equations (5) and (6) together constitute the rare events approximation. The results for the union of events are derived in Appendix A from which it is inferred that self-similar temporal scaling (temporal scale invariance) is only a useful concept for individual avalanche paths with very long waiting times between events. For the avalanche areas as a whole, the time scale of applicability is on the order of a few hours.

In this section, the formulation for the intersection of events (equation 6) is developed based on independence of the individual paths. The object is to assess whether the frequency power spectrum has  $1/f$  dependence over any significant frequency range for the collection of avalanche paths in the two areas.

Assuming completely random events, the probability density function for an avalanche path takes the form of equation (4) and the auto-correlation function is easily calculated as:

$$c(t, \tau_i) = \exp(-t / \tau_i) \quad (7)$$

For a single avalanche path, the frequency power spectrum,  $S(f)$ , is of the same form as equation (2) with the replacement:  $\tau_i \rightarrow \tau_0$ . From Figs. 24.7 and 24.8, the analysis suggests that the probability density function for  $\tau_i$  in both areas is log-normal. Thus, a distribution of mean waiting times is assumed such that:

$$\int_0^{\infty} \rho(\tau) d\tau = 1 \quad (8)$$

In equation (8) the subscript on  $\tau$  is dropped and a continuous distribution of waiting times  $\rho(\tau)$  is assumed.

Since the individual avalanche paths are separate and disjoint, I assume no correlations between them and the power spectrum for an avalanche area is given by the weighted sum from the individual avalanche paths (Van der Ziel 1950; Montroll

and Shlesinger 1983; Schroeder 1990; Jensen 1998). From equation (2), the power spectrum for an avalanche area then becomes:

$$S(f, \tau_0) = 4 \operatorname{Re} \int_0^{\infty} c(t, \tau_0) e^{2\pi i f t} dt = 4\tau_0 / [1 + (2\pi f \tau_0)^2] \quad (9)$$

From Figs. 24.7 and 24.8, I suggest that is reasonable to consider  $\rho(\tau)$  as log-normal. One may also derive this by considering  $\tau = 1/P$  as the inverse of a probability per unit time following *Montroll and Shlesinger* (1983). Assuming all the  $\tau_i$  are independent as in equation (6), then

$$\log(\tau) = \log(1/P) = \sum_{i=1}^N \log(1/p_i) = \sum_{i=1}^N \log(\tau_i) \quad (10)$$

In equation (10),  $P$  represents the probability per unit time that a ‘grand success’ occurs meaning that the system of avalanche paths is represented in series such that a ‘grand success’ requires that all are active. Since each  $p_i$  is a random variable then

so is  $\tau_i$  and equation (10) follows from the independence assumption of equation (6). From the central limit theorem if  $N$  becomes large, and the first and second moments exist, then it is implied that  $\log(\tau)$  has a normal distribution or  $\tau$  has a log-normal one, as suggested by the data. Now let  $\bar{\tau}$  be the mean value of  $\tau$ . From Table 24.1, the values of  $\bar{\tau}$  are 43 days for Bear Pass and 94 days for Kootenay Pass or 54 days and 96 days from Table 24.3 if only natural events are considered.

The log-normal distribution has the form:

$$\rho\left(\frac{\tau}{\bar{\tau}}\right) d\tau = \frac{\exp\{-[\log(\tau/\bar{\tau})]^2 / 2\sigma_{\ln(\tau)}^2\}}{(2\pi\sigma_{\ln(\tau)}^2)^{1/2}} \frac{d\tau/\bar{\tau}}{\tau/\bar{\tau}} \quad (11)$$

The power spectrum is then:

$$S(f) = 4\bar{\tau} \int_0^{\infty} \frac{(\tau/\bar{\tau}) \exp(-[\log(\tau/\bar{\tau})]^2 / 2\sigma_{\ln(\tau)}^2) d(\tau/\bar{\tau})}{(2\pi\sigma_{\ln(\tau)}^2)^{1/2} (\tau/\bar{\tau}) [1 + (2\pi f \tau)^2]} \quad (12)$$

Some important limits from the integration of (12) were derived by Montroll and Shlesinger (1983).

If  $\sigma_{\ln(\tau)}^2 \rightarrow \infty$  then



$$S(f) \sim \frac{4\bar{\tau}}{4\pi f \bar{\tau} \sigma_{\ln(\tau)}} \left( \frac{\pi}{2} \right)^{1/2} \quad (13)$$

and a  $1/f$  spectrum results.

If  $f \rightarrow 0$  then

$$S(f) \sim 4\bar{\tau} \exp\left(\frac{1}{2} \sigma_{\ln(\tau)}^2\right) \quad (14)$$

and white noise (constant power spectrum) results at low frequency similar to the low frequency limit of the power spectrum for an individual path as in equation (2).

If  $f \rightarrow \infty$  then

$$S(f) \sim \frac{4\bar{\tau}}{(2\pi f \bar{\tau})^2} \exp\left(\frac{1}{2} \sigma_{\ln(\tau)}^2\right) \quad (15)$$

and Brownian noise ( $1/f^2$ ) spectrum results similar to the high frequency limit for a single path (equation 2).

These results (equations 13-15) are related to those of Van der Ziel (1950), Schroeder (1990) and Jensen (1998). They all assumed (on an ad-hoc basis) a wide (high variance) uniform distribution of  $\tau$  to produce a  $1/f$  spectrum over a wide range of frequencies with  $1/f^2$  behaviour at high frequencies and white noise at low frequencies.

Since the spectrum of equation (12) goes from white noise at low frequency to  $1/f^2$  at high frequency it is suggested that there may be a range of intermediate frequencies in which the spectrum mimics  $1/f$  noise (also called flicker noise or pink noise). The other possibility for  $1/f$  noise arises if  $\sigma_{\ln(\tau)}^2$  is very large. However, for the present analysis, the values in Tables 24.2 and 24.4 show that such is not the case.

Montroll and Shlesinger (1983) performed the asymptotic integration of the power spectrum given by equation (12). If  $|\ln(2\pi f \bar{\tau})| < 2\sigma_{\ln(\tau)}^2$  then  $S(f)$  has a simplified log-normal form:

$$S(f) = \frac{C_0}{\sigma_{\ln(\tau)} \pi f} \left( \frac{\pi}{2} \right)^{1/2} \exp \left[ -\frac{1}{2} \left( \frac{\ln \{2\pi f \bar{\tau}\}}{\sigma_{\ln(\tau)}} \right)^2 \right] \quad (16)$$

where

$$C_0 = \left( \left[ 1 - \frac{1}{2} \left( \frac{\pi}{2\sigma_{\ln(\tau)}} \right)^2 + \frac{5}{8} \left( \frac{\pi}{2\sigma_{\ln(\tau)}} \right)^4 \right] + \dots \right).$$

From Tables 24.1 and 24.2, the condition that equation (16) applies is:

$$f < 0.033 = 1/30(\text{day})^{-1} \text{ (Bear Pass) and } f < 0.169 = 1/5.9(\text{day})^{-1} \text{ (Kootenay Pass).}$$

These calculations show the importance of the mean waiting time,  $\bar{\tau}$ , and the variance,  $\sigma_{\ln(\tau)}^2$ , in order for the spectrum to have the simple log-normal form of equation (16). Kootenay Pass has higher mean and variance to greater possibility for the log-normal form. For natural events only, the above values are replaced by  $f < 0.027 = 1/37(\text{day})^{-1}$  (Bear Pass) and  $f < 0.012 = 1/83(\text{day})^{-1}$  (Kootenay Pass) and the effect of the reduced variance considering only natural events at Kootenay Pass is evident.

From the form of equation (16), it is clear that if the argument of the exponential is  $\ll 1$  then the spectrum approximates  $1/f$  dependence. Following Montroll and Shlesinger (1983) taking the log of equation (16) gives a form which can be analysed for approximating  $1/f$  dependence:

$$\ln[S(f)] = -\ln(f) - \frac{1}{2}[(\ln f)^2 / \sigma_{\ln(\tau)}^2] - \ln(C_1) \quad (17)$$

where  $C_1$  is a constant. If the spectrum is to be  $1/f$  then the middle term must be much smaller than the first term so that only the first and last terms are present. Let  $f = \exp(n_0)$  then equation (17) becomes:

$$\ln[S(f)] = -n_0 - \frac{1}{2}(n_0^2 / \sigma_{\ln(\tau)}^2) - \ln(C_1) \quad (18)$$

If the second term is to be less than a small fraction,  $\theta$ , of the first term (Montroll and Shlesinger 1983) then

$$|n_0| \leq 2\theta\sigma_{\ln(\tau)}^2 \quad (19)$$

and  $S(f)$  approximates a  $1/f$  distribution for  $4\theta\sigma_{\ln(\tau)}^2 + 1$  e-folds to within a relative error  $\theta$ . If  $\theta = 0.1$  then, within relative error of 10% and with values from Table 24.2, this amounts to 1.4 e-folds for Bear Pass and 1.9 e-folds for Kootenay Pass. If only natural events are considered from Table 24.4, the spectra from both areas follow  $1/f$  with a 10% error for about 1.4 e-folds.

Generally, to claim scale invariance ( $1/f$  dependence in this case) for a critical geophysical system (Jensen 1998; Korvin 1992) it must be demonstrated at least two and preferably several orders of magnitude. From the analysis here, I conclude that this condition is not fulfilled. The principal reason is that the variance,  $\sigma^2_{\ln(\tau)}$ , is not large enough for either system of avalanche paths at Bear Pass and Kootenay Pass. For the avalanche areas here, the waiting times for individual paths vary only by about two orders of magnitude. From equation (17), it is clear that if  $\sigma^2_{\ln(\tau)}$  is large enough then  $1/f$  dependence results and it is the only physical parameter that matters for the present analysis once the frequency requirement for satisfying equation (16) is attained.

## 10 Discussion and Conclusions

The postulates of self-organized criticality include lack of both characteristic length and time scales for a series of events. I have shown previously (McClung 2003(a)) that for snow slab avalanches, there is always a characteristic length scale associated with the events, namely, the depth to the weak layer determines a characteristic size and the avalanches are nearly all the same size during any short time frame. In contrast, for the definition of self-organized criticality provided by Bak *et al.* (1987) a spectrum of sizes is expected. In addition, even if the characteristic sizes of slab avalanches are analysed (McClung 2003(a)) they do not follow a power law distribution over any significant range of sizes as suggested for SOC. The limited range of slab thickness combined with the fact that the probability plots of slab depth, length and width all suggest they follow log-normal probability density functions suggests that scale invariance with respect to size is not followed.

Snow avalanches and earthquakes constitute two of a very few entities in solid earth surface geophysics with a high enough frequency of events for analysis of time dependence for testing the definition of a critical system. For the mountain slope hazards (debris flows, landslides, rock avalanches, snow avalanches, rock falls), snow avalanches and rock falls (due to high frequency of events) are the only ones with a reasonable chance of testing the postulated scale invariance of the power spectrum in the frequency domain required for an SOC system. A major object of the present chapter has been to provide a fairly comprehensive test of whether or not scale invariance exists in the frequency domain for snow avalanche events. Previously, (McClung 2003(b)), I analysed time arrival data for Bear Pass and Kootenay Pass, and I found consistent results that a characteristic time scale is present. In this chapter, a companion analysis of waiting time between avalanches is provided. The results of the present analysis are consistent with the previous results. Namely, it is unlikely that frequency scale invariance is present in the waiting time analysis over any significant time or frequency range. The results with respect to waiting time here rest on the assumption that avalanches can be regarded as Poisson

events (McClung 1999) so any conclusions can be regarded as within the framework of model assumptions. Thus, if the assumptions of the model are incorrect, then so will the conclusions be incorrect or inconclusive. Table 24.5 contains a summary of the mathematical results of this chapter in comparison with the postulates of SOC based on the analysis presented.

**Table 24.5.** Postulates of SOC compared with results from field measurements and observations of snow avalanches.

Postulate of SOC	Snow avalanche results
No characteristic size	Loose snow avalanches are nearly all the same size for the same snow pack conditions.
No characteristic size or scale invariant size distribution	Slab avalanches have a characteristic size (or length scale) $D$ . The cumulative distribution for $D$ is not scale invariant over any significant range.
Scale invariant frequency power spectrum (no characteristic time scale)	Slab avalanche arrivals viewed as a system in an avalanche area have a characteristic relaxation time $\tau_0$ .
Scale invariant frequency power spectrum (e.g. $\sim 1/f$ )	Slab avalanche waiting time may have power spectrum approximately proportional to $1/f$ for about 1 - 2 e-folds with 10% error.

When the results of the present analysis are combined with my previous results, I suggest that snow avalanches (both loose and slab avalanches) do not conform to the mathematical formalism about self-organized criticality postulated by Bak et al. (1987). For slab avalanches neither with size nor frequency invariance applies.

In spite violation of the mathematical postulates of SOC to snow avalanches and other related phenomena such as sandpile avalanches, it is an undeniable achievement of Per Bak to attempt to highlight the possible important commonalities of geophysical events such as the mountain slope hazards and earthquakes. There is definitely something critical about the release of a snow slab avalanche, a rock avalanche, a flake type (e.g. quick clay) landslide or an earthquake caused by fracture propagation along rock joints. They all exhibit slow build-up and evolution until catastrophic, rapid failure occurs (sometimes under very small influences). For example, dry snow slabs sometimes are observed when air temperatures change without additional mechanical load being applied. Further, the mountain slope hazards and earthquakes, all may have strain-softening behaviour (see Figure 24.3) during failure and prior to fracture which suggests they can be regarded as due to failure of quasi-brittle materials with implied fracture mechanical size effects and important length scales prior to self-propagating fractures as outlined by Bažant and Planas (1998) and Bažant et al. (2003) for the snow slab.

The presence of a time scale in snow avalanche arrivals or waiting times is consistent with human knowledge in relation to avalanche forecasting. Most natural avalanches happen during or immediately after snow storms with failure on weak layers in the storm snow. Persistent weak layers such as surface hoar, faceted snow or depth hoar [see McClung and Schaerer (2006) for more information] can result in large avalanches sometimes months after the weak layers have formed. However, upon application of perturbations of snow loading by storms or blowing snow, nearly all natural avalanches happen during or immediately (for example within a day or two) after load application.

The definition of a time scale (e.g. return period) or waiting time for individual avalanche paths is the basis for land-use planning in avalanche terrain. It depends importantly on terrain characteristics and the snow climate (McClung 2003(c)). Of course, for either a return period or a wait time, there is no information about ‘when’ events will occur. Only the statistical properties of many avalanches including their size and frequency give rough ideas about how often, on average, events are expected and expected sizes. Bak [1996] emphasizes this statistical or probabilistic character as an important characteristic of a critical system.

The presence of a time scale in arrivals or wait times for natural, open, non-equilibrium systems is probably more the rule (Korvin 1992) than the exception. However, Bak et al. (1987) postulated for SOC that lack of a time scale is necessary. A more relaxed criterion for the frequency power spectrum,  $S(f)$ , such as a decay from Brownian noise at high frequency to white noise at low frequency might be a more suitable common property for prospective geophysical critical systems in nature. A scale invariant form such as  $1/f$  or  $1/f^\beta$  seems to require a very high variance in waiting time to evolve, particularly if scale invariance is to be present over a significant frequency range such as more than one order of magnitude. Since it is not possible (Van der Ziel 1950; Montroll and Schlesinger 1983) for a form like  $1/f$  to be present over all frequencies (e.g. the spectrum diverges at zero frequency) then natural variations in frequency must vary over at least 3, and perhaps 4, orders of magnitude in order to achieve scale invariance over two orders of magnitude. For snow avalanches and the mountain slope hazards, such wide variations are not easily observed or verified.

From observations, there are two distinct cases that characterize time instability (McClung 2002(b)) for an avalanche area which I suggest may be classed as an open, non-equilibrium system. Most of the time, conditional instability prevails, whereby additional energy must be supplied to produce fractures and trigger avalanches. This situation produces occasional avalanches at isolated locations which are difficult to locate. The difficulty in locating potential fracture points is the primary source of fatal accidents in back-country travel which constitute more than 90% of avalanche deaths in North America and Europe (McClung and Schaerer 2006). The second type is: absolute instability, whereby the system reaches critical instability in many avalanche paths over considerable spatial extent (sometimes over an entire mountain range) and there are widespread natural releases. The latter

produces a ‘burst’ of activity, called an avalanche cycle. Within the time frame of a winter, conditional instability is the rule (prevailing most of the winter) and absolute instability is the exception, often being observed for only a few hours in an avalanche area during a winter (McClung 2002(b)).

The time scale of several hours for self-similar clustering of wait times (several hours) for events at Bear Pass and Kootenay Pass (derived in Appendix A) matches the observed time scale for absolute instability suggested by McClung (2002(b)). Rare winters with high losses and many large avalanches, such as 1999 in the European Alps of Switzerland, Austria and France are observed when a persistent weak layer forms early in the winter (in this case over nearly the entire Alps) with overloading by heavy snow falls later to generate large avalanches from the deeply buried weak layer over a storm time scale of one to several days. In this sense, the two cases of instability for snow avalanches provide at least a qualitative match to Per Bak’s (Bak 1996, p. 59 - 62] description of a non-equilibrium, open system and the expected character of events. The absolute instability situation corresponds to evolution of the system making large transformations (avalanche cycles) over short time scales (hours for avalanche areas; days for a mountain range) as Bak also suggests for self-organized criticality. In contrast, the conditional instability represents a relative ‘stasis’ with isolated events in time and space. The clustering of events separated by periods of stasis is analogous to punctuated equilibrium described by Bak (1996) as a characteristic of complex non-equilibrium systems some of which exhibit  $1/f$  noise and fractal behaviour in relation to event size to comprise SOC. There is no argument about the fact that a system of snow avalanche paths displays the signals of punctuated equilibrium. However, the analysis in this chapter combined with field observations suggests that the  $1/f$  noise and fractal behaviour are absent.

Large transformations over short time scales are occasional features of scientific thought and Per Bak may have been an architect of such a transformation. For snow avalanches, the mathematical details suggest snow avalanches are non-critical with respect to the original ideas about SOC. However, human experience supported by field measurements suggests the physical ideas do have merit. In the midst of an avalanche cycle, the notion of an average waiting time or return period has very limited relevance. Instead, we can expect Bak’s time-scale-free dependence to prevail. In time, avalanche events (see Figure 24.5) consist of clusters, intermittencies and bursts (Birkeland and Landry 2002; McClung 2003(b)). Avalanche forecasting (McClung 2002 (a)) is concerned with the specifics of predicting rapid changes in space and time relative to a given triggering energy level so forecasters implicitly follow the principles of punctuated equilibrium which is one of descriptive aspects of SOC. However, such forecasting was developed on the basis of practical experience long before SOC was proposed. The unstable equilibrium which characterizes snow slab avalanche release (Figure 24.2) suggests a different kind of criticality which is highly significant to humans and which is shared by earthquake, rock avalanche and rapid landslide release. The prediction problem reduces to much the same thing for either type of criticality: statistical and probabilistic rather than

deterministic. However, the details of criticality for the snow slab are more likely rooted in fracture mechanics (McClung 1979) instead of scale invariance.

## Acknowledgements

This research was funded by Canadian Mountain Holidays and the Natural Sciences and Engineering Research Council of Canada. I am very grateful for this support. I am also thankful for data from Bear and Kootenay Passes which were provided by the British Columbia Ministry of Transportation and Highways. The suggestion to analyse wait times was provided by Professor Garry Clarke. The mathematical brilliance of Elliott Montroll made the wait time analysis possible.

## References

- Bak, P. (1996), How nature works, the science of self-organized criticality, *Springer-Verlag New York Inc.*, New York, U.S.A., 212 pp.
- Bak, P., Tang, C. and K. Wiesenfeld (1987), Self-organized criticality: An explanation of  $1/f$  noise, *Physical Review Letters*, 59, 381 - 384.
- Bažant, Z.P. and J. Planas (1998), Fracture and size effect in concrete and other quasibrittle materials, *CRC Press*, Boca Raton, USA, 616 pp.
- Bažant, Z.P., G. Zi and D. McClung (2003), Size effect and fracture mechanics of the triggering of dry snow slab avalanches, *J. Geophys. Res.*, Vol. 108, No. B2, 2119, doi:10.1029/2002JB001884.
- Birkeland, K.W. and C.C. Landry (2002), Power laws and snow avalanches, *Geophys. Res. Lett.*, 29(11), 49-1–49-3.
- Benjamin, J.R. and C. A. Cornell (1970), Probability, statistics and decision for civil engineers, *McGraw-Hill Inc*, New York, U.S.A., 684 pp.
- Ito, K. (1995), Punctuated equilibrium model of biological evolution is also a self-organized critical model of earthquakes, *Phys. Rev. E* (52): 3232.
- Jensen, H.J. (1998) Self-organized criticality, emergent complex behaviour in physical and biological systems, Cambridge Lecture Notes in Physics 10, *Cambridge University Press*, Cambridge, U.K., 153 pp.
- Korvin, G. (1992), Fractal models in the earth sciences, *Elsevier*, Amsterdam, 396 pp.
- McClung, D.M. (1979), Shear fracture precipitated by strain-softening as a mechanism of dry slab avalanche release, *J. Geophys. Res.*, 84(B7), 3519-3526.
- McClung, D.M. (1999), The encounter probability for mountain slope hazards, *Canadian Geotechnical Journal*, Vol. 36, No. 6, 1195 -1196.
- McClung, D.M. (2002(a)), The elements of applied avalanche forecasting part I: the human issues, *Natural Hazards*, Vol. 26, No. 2, 111-129.
- McClung, D.M. (2002(b)), The elements of applied avalanche forecasting part II: the physical issues and rules of applied avalanche forecasting, *Natural Hazards*, Vol. 26, No. 2, 131-146.
- McClung, D.M. (2003(a)) Size scaling for dry snow slab release, *J. Geophys. Res.*, Vol. 108, No. B10, 2465, doi:1029/2002JB002298.
- McClung, D.M. (2003 (b)) Time arrival of slab avalanche masses, *J. Geophys. Res.* Vol. 108, No. B10, 2466, doi:10.1029/2002JB002299.

- McClung, D.M. (2003(c)) Magnitude and frequency of avalanches in relation to terrain and forest cover, *Arctic, Antarctic and Alpine Res. Vol. 35, No.1*, 82-90.
- McClung, D.M. (2005), Dry slab avalanche shear fracture properties from field measurements, *J. Geophys.Res.*, *110*, F04005, doi:10.1029/2005JF000291.
- McClung, D. and P. Schaerer (2006), The Avalanche Handbook, 3<sup>rd</sup> Edition, *The Mountaineers Books*, Seattle, Wash., 342 pp.
- McCormick, N.J. (1981), Reliability and risk analysis, methods and nuclear power applications, *Academic Press, Inc.*, Boston, U.S.A., 446 pp.
- Montroll, E.W. and M.F. Shlesinger (1983), Maximum entropy formalism, fractals, scaling phenomena, and  $1/f$  noise: a tale of tails, *Journal of Statistical Physics*, *Vol. 32, No.2*, 209-230.
- Schroeder, M. (1990), Fractals, chaos, power laws, minutes from an infinite paradise, *W.H. Freeman and Company*, New York, 429 pp.
- Shlesinger, M.F. and B.D. Hughes (1981), Analogs of renormalization group transformations in random processes, *Physica 109A*, 597-608.
- Van der Ziel, A. (1950), On the noise spectra of semi-conductor noise and flicker effect, *Physica XVI, no. 4*, 359-372.



## Appendix A : Implication of Rare Events Approximation for a Set of Disjoint Avalanche Paths for the Union of Events

Consider a set of avalanche paths,  $A_1, A_2, \dots, A_N$  which are disjoint with rare events assumed not to overlap in time. Under the rare events approximation (McCormick 1981;p.16): the probability of the union of events is given by:

$$P(A_1 \cup A_2 \cup A_3 \cup \dots A_N) \approx \sum_{i=1}^N P(A_i) \quad (A1)$$

As a guideline, the rare events approximation holds when  $P(A_i) \leq 0.1$ .

For a random process with a sequence of events, the waiting time density between events is  $\psi(t)$  and  $\psi(t)dt$  is the probability that an event occurs in the interval  $(t, t + dt)$  with the most recent event at  $t = 0$ . Assuming the rare events approximation holds, for  $N$  avalanche paths partitioned in the sample space of an avalanche area, the probability in terms of the normalized density is:

$$\psi(t)dt = dt \sum_{i=1}^N a_i \psi_i(t) = \frac{dt}{N} \sum_{i=1}^N \psi_i(t) \quad (A2)$$

$$\text{where } \sum_{i=1}^N a_i = 1; a_i = \frac{1}{N}.$$

In Laplace transform space, for an individual path,  $i$ ,

$$\psi_i^*(\varepsilon) = \int_0^{\infty} e^{-\varepsilon t} \psi_i(t) dt \quad (A3)$$

For completely random Poisson events, the waiting time between events is taken to be exponentially distributed from equation (4):

$$\psi_i(t) = \frac{1}{\tau_i} e^{-\frac{t}{\tau_i}}; t \geq 0 \quad (A4)$$

where  $\tau_i$  is the mean waiting time between events for avalanche path  $i$ . From (A3),

$$\psi_i^*(\varepsilon) = \frac{1}{1 + \tau_i \varepsilon} \quad (A5)$$

The probability density for the time of occurrence of  $n$ th event on path  $i$  is:

$$\begin{aligned}\psi_{i,n}(t) &= \int_0^\infty \psi_{i,n-1}(t-\tau) \psi_i(\tau) d\tau, \\ \psi_{i,n}^*(\varepsilon) &= \int_0^\infty \exp(-\varepsilon t) [\psi_{i,n}(t)] dt\end{aligned}\quad (\text{A6})$$

For a process started at  $t = 0$ ,

$$\psi_{i,n}^*(\varepsilon) = [\psi_i^*(\varepsilon)]^n \quad (\text{A7})$$

For an individual avalanche path, from (A4) and (A6), the time to the  $n$ th event is given by the gamma distribution (Benjamin and Cornell 1970):

$$\psi_{i,n}(t) = \frac{1}{\tau_i^n (n-1)!} t^{n-1} \exp(-t/\tau_i) \quad (\text{A8})$$

From (A5) and (A7),

$$\psi_{i,n}^*(\varepsilon) = \frac{1}{(1 + \tau_i \varepsilon)^n} \quad (\text{A9})$$

If  $\tau_i \varepsilon \gg 1$  (or  $t/\tau_i \ll 1$ ), then the scale invariant form:

$$\psi_n^*(\varepsilon) \sim \frac{1}{N} \sum_{i=1}^N \frac{1}{(\tau_i \varepsilon)^n} = \frac{1}{\varepsilon^n \tau_{inv}^n} \quad (\text{A10})$$

results where

$$\tau_{inv}^n = \left( \frac{1}{N} \sum_{i=1}^N \frac{1}{\tau_i^n} \right)^{-1}. \quad (\text{A11})$$

Direct integration of (A2) gives the cumulative distribution function:

$$P_n(t) = \frac{t^n}{n! \tau_{inv}^n} \left[ 1 - \left( \frac{n}{n+1} \right) \frac{t}{\tau_{inv}} + O\left( \frac{t^2}{\tau_{inv}^2} \right) - \dots \right] \quad (\text{A12})$$

and if  $nt \ll (n+1)\tau_i$ ,  $i = 1, N$  then the scale invariant form:

$$P_n(t) = \frac{t^n}{n! \tau_{inv}^n} \quad (\text{A13})$$

results for the avalanche area under the rare events approximation.

For Bear Pass (73 paths) and Kootenay Pass (37 paths), values for  $\overline{\tau_{inv}}$  are 15 and 14 days respectively. These results suggest that a finite time scale from exponential decay will be associated with the process unless the minimum of the average wait times is large. Given the minimum of the mean waiting times,  $\tau_i$ , for individual paths at Kootenay Pass (3 days) and Bear Pass (2 days), the power law (scale invariant) behaviour implied by (A13) applies only for times less than a few hours respectively for either pass. If the rare events approximation does not hold ( $t > 0.1\tau_i$ ) then a finite time scale will be associated with the process but the analysis would be intractable.

For a Poisson process, there is no memory of past events so that time  $t = 0$  can be taken at any arbitrary time of the most recent event. The time estimated above for scale invariant time dependence (or self-similar clustering of event wait times) for the avalanche areas is less than storm durations during which many avalanche events are observed over less than half a day. If time  $t = 0$  is taken near the beginning of a storm, then there will be time overlap between events and the rare events approximation will not hold. For Bear Pass, the probability of one event in a time of less than 4 hours is about  $1/76$  and the probability of two events is about  $1/2825$  from (A13). These results suggest that self-similar clustering or scale invariant behaviour is of short duration for these avalanche areas. If the minimum wait time was on the order of months or years, instead of days, different conclusions would arise.

In general, self-similar clustering of event waiting times for an individual avalanche path,  $i$ , will occur if  $t \ll \tau_i$  or  $\tau_i \varepsilon \gg 1$ . In this case, the decay of  $\psi_{i,n}(t)$  will be slow enough that no finite time scale will occur in the analysis and a scale invariant form  $\psi_{i,n}(t) \sim t^{n-1}$  [see equation (A8)] develops. For  $t \ll \tau_i$ ;  $i = 1, N$  (or  $t < 0.1\tau_i$  approximately) the rare events approximation for the union of events is expected to apply and each term in the finite series (A1) has the scale invariant form  $P_{i,n} \sim t^n$  so self-similar clustering of event waiting times arises as in (A13) for a collection of paths.

If the mean waiting time  $\tau_i \rightarrow \infty$ , then no characteristic time appears and self-similar clustering of event times for an avalanche path occurs. This was shown by Shlesinger and Hughes (1981) and Montroll and Shlesinger (1983) for temporal scaling of Poisson events. From (A9), if  $\tau_i$  is finite, then Taylor series expansion around  $\varepsilon \rightarrow 0$  (for long times) gives:

$$\psi_{i,n}^*(\varepsilon) = 1 - n\varepsilon\tau_i + \frac{1}{2!}(n)(n+1)\varepsilon^2\tau_i^2 - \dots \quad (\text{A14})$$

Equation (A14) shows that a finite time scale will be contained in the process at long times unless the mean waiting time is infinite. For  $\tau_i \rightarrow \infty$  then  $\psi_{i,n}^*(\varepsilon), \psi_{i,n}(t)$

both take scale invariant form.

For the events at Bear Pass (73 paths) and Kootenay Pass (37 paths), the mean of the waiting times between events for all paths taken together are 43 and 94 days (or 54 and 96 days for natural events only) respectively. McClung (2003(a)) showed from calculation of the auto-correlation function from time series of events at Bear Pass and Kootenay Pass that an e-folding time of about 1 day is associated with arrivals which suggests a short mean time scale for event occurrence.

The differences in time scales (between the mean waiting values and the e-folding arrival time) may be related to the lack of applicability of the rare events approximation for Bear Pass and Kootenay Pass time arrival analyses of McClung (2003(a)). It would be impossible to record events exactly when they occur over many paths even though they rarely, if ever, overlap in time if the observation time is short enough. The occurrence data in the analysis of McClung (2003(a)) were lumped into 12 hour time intervals. This would imply time overlap during storms in the analysis so the rare events approximation would not apply. Instead, the highest frequency paths (minimum wait times) would control the time scale for the analysis. Thus, minimum rather than average wait times would apply and the short finite relaxation time scale suggested by the time series seems reasonable since it is comparable to minimum wait times and storm durations at Bear Pass and Kootenay Pass.

From the analysis above, I conclude that self-similar clustering of event waiting times for the avalanche areas of Bear Pass and Kootenay Pass is unlikely except for short periods. The fundamental reason is that paths of short mean waiting time are present which limits the time scale of applicability as derived from the approximations needed. In turn, this suggests that a finite time scale associated with exponential decay should be associated with the process for any longer time duration. The short time 'bursts' of avalanche activity associated with large avalanche cycles which occur about once a year in avalanche areas may be related a lack of waiting time scale for short periods. I have termed such short time 'bursts' absolute instability (McClung 2002 (b)).

For individual avalanche paths, the average delay between events on paths ranges up to about 540 days at Kootenay Pass (for all events and natural events) and 275 days at Bear Pass (385 days for natural events). For avalanche paths with such long delay times, self-similar clustering of event times (or scale invariant behaviour) can be expected over fairly substantial times which exceed storm durations.