Some words on the analytic hierarchy process and the provided ArcGIS extension ‘ext_ahp’

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The extension ext_ahp has been tested with ArcGIS 9 running on a WindowsXP operating system.

Short introduction to the principles of the Analytic Hierarchy Process (AHP)

A decision is a result of a comparison of one or more alternatives with respect to one ore more criteria that we consider relevant for the decision. Among these relevant criteria, we consider some as more and some as less important which is equal to assigning weights to the criteria according to their relative importance. The Analytic Hierarchy Process (AHP) was introduced by SAATY (1977) and is a very popular means to calculate the needed weighting factors. This is done by the help of a preference matrix where all identified relevant criteria are compared against each other with reproducible preference factors.

AHP process description

All criteria/factors which are considered relevant for a decision are compared against each other in a pair-wise comparison matrix which is a measure to express the relative preference among the factors. Therefore numerical values expressing a judgement of the relative importance (or preference) of one factor against another have to be assigned to each factor. Since it is known from psychological studies that an individual cannot simultaneously compare more than 7 ± 2 elements, SAATY (1977) and SAATY & VARGAS (1991) suggested a scale for comparison consisting of values ranging from 1 to 9 which describe the intensity of importance (preference/dominance). A value of 1 expresses “equal importance” and a value of 9 is given for those factors having an “extreme importance” over another factor (Table 1).

Table 1: Example scale for comparisons (SAATY & VARGAS, 1991)

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance of one factor over another</td>
</tr>
<tr>
<td>5</td>
<td>Strong or essential importance</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values</td>
</tr>
<tr>
<td>Reciprocals</td>
<td>Values for inverse comparison</td>
</tr>
</tbody>
</table>
Example

Table 2 shows a simple comparison matrix of order 3 where 3 criteria C1, C2 and C3 are compared against each other. In the direct comparison of the criteria C1 and C3, for example, criterion C1 has been regarded strongly more important than C3, hence a value of 5 has been assigned to the corresponding matrix position. The transpose position automatically gets a value of the reciprocal value, in this case 1/5 which equals 0.2.

Table 2: Example of a pair-wise comparison matrix describing the relative preference among the identified criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>C2</td>
<td>0.25</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>C3</td>
<td>0.2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Next, the assigned preference values are synthesised to determine a ranking of the relevant factors in terms of a numerical value which is equivalent to the weights of the factors. Therefore the eigenvalues and eigenvectors of the square preference matrix revealing important details about patterns in the data matrix are calculated.

The above square matrix of order three gives three eigenvalues with which three eigenvectors – each having three vector components - can be calculated. It is regarded sufficient to calculate only the eigenvector resulting from the largest eigenvalue since this eigenvector contains enough information to provide – by its eigenvector components - the relative priorities of the factors being considered (SAATY & VARGAS 1991).

Of course, the values of the pair-wise comparison matrix will normally be well considered and not set arbitrarily. However, people’s feelings and preferences remain inconsistent and intransitive and may then lead to perturbations in the eigenvector calculations. Such inconsistencies might be of the form that a factor A_i being preferred over another factor A_j with A_j being pre-ferred over a factor A_k is not preferred over A_k (A_i must be preferred over A_k in this case). Therefore SAATY (1977) provided the consistency ratio CR which is a single numerical index to check for consistency of the pair-wise comparison matrix. It is defined as the ratio of the consistency index CI to an average consistency index RI, thus

\[
CR = \frac{CI}{RI}
\]

Table 3: Values for RI (SAATY & VARGAS, 1991; with n = order of matrix)

<table>
<thead>
<tr>
<th>n</th>
<th>RI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
</tr>
<tr>
<td>7</td>
<td>1.32</td>
</tr>
<tr>
<td>8</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Following the illustrations of SAATY (1977) who calculated RI values up to a matrix order of 15, matrices with an order greater than 8 have an RI order of magnitude value of about 1.45.

The consistency index CI can be directly calculated from the preference matrix with
CI = \frac{\lambda_{\text{max}} - n}{n - 1} \quad (2)

where

\lambda_{\text{max}}: \quad \text{greatest eigenvalue of preference matrix}

n: \quad \text{order of matrix}

Based on expert knowledge and experience, SAATY & VARGAS (1991) recommend a revision of the preference matrix if the consistency ratio CR exceeds a value of 0.1.

**Prioritisation of alternatives**

The AHP also allows a prioritisation of decision alternatives. Let us consider two alternatives A1 and A2 and three identified criteria C1, C2 and C3 whose preferences against each other are already established in Table 2. The criteria weights calculated from the eigenvector of the largest eigenvalue of this criteria preference matrix are given in Table 4.

Table 4: Criteria weights resulting from the preference matrix (see Table 2)

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.687</td>
</tr>
<tr>
<td>C2</td>
<td>0.127</td>
</tr>
<tr>
<td>C3</td>
<td>0.186</td>
</tr>
</tbody>
</table>

In the following step, the two alternatives have to be compared with respect to the relevant criteria. The calculated weights are again calculated from the eigenvectors of the comparison matrices.

**Criterion C1:**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A1</th>
<th>A2</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>5</td>
<td>0.83</td>
</tr>
<tr>
<td>A2</td>
<td>0.2</td>
<td>1</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Criterion C2:**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A1</th>
<th>A2</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>A2</td>
<td>0.33</td>
<td>1</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Criterion C3:**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A1</th>
<th>A2</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>0.166</td>
<td>0.14</td>
</tr>
<tr>
<td>A2</td>
<td>6</td>
<td>1</td>
<td>0.85</td>
</tr>
</tbody>
</table>

The of the available alternatives is then calculated as follows

Alternative A1: \(0.687 \cdot 0.83 + 0.127 \cdot 0.75 + 0.186 \cdot 0.14 = 0.69\)

Alternative A2: \(0.687 \cdot 0.17 + 0.127 \cdot 0.25 + 0.186 \cdot 0.85 = 0.31\)

Since alternative A1 gets a higher score, it should be preferred over alternative A2.
The AHP extension ‘ext_ahp’ in ArcGIS

Accessing the functionality of ‘ext_ahp.dll’

To make the extension run it is necessary to copy the extension (ext_ahp.dll) and the library EigenUtl.dll on your computer. A good choice for a destination folder is the ‘ArcGIS\Bin’ folder. In case you choose a different path please note however, that EigenUtl.dll which returns the eigenvalue/eigenvector results (see Marinoni 2004) must be put into a folder which is set on the Windows™ path environment (e.g. into the Windows system directory). Having copied the files, perform the following steps:

1. launch ArcGIS

![Customize](image)

2. go to the Tools menu, choose Customize
3. push the ‘Add from file’ button
4. browse for the extension ‘ext_ahp.dll’ and choose ‘Ok’
5. go to the ‘Commands’ tab. In the ‘Categories’ list box choose ‘Developer Samples’
6. Choose ‘AHP’ in the ‘Commands’ list box
7. Click on the list item which is labeled ‘AHP’ and move this item by dragging it somewhere onto the ArcMap environment.
8. Push the ‘Close’ button. The extension is ready for use.

If you decided to place an additional button (point 7, see above) your ArcMap environment could look like this:

![ArcMap environment](image)
Using the extension:

Push the ‘AHP’ button. A window appears which lets you define all criteria which you consider relevant for your decision. Please note: The extension expects classified raster datasets. Therefore only integer rasters will appear as list items in the classified map raster layers list box. The user must take care that all rasters are classified within the same class value range (no checking is performed if the classified criteria rasters have the same class range). Click on a list item and the push the button to define it as a criterion which will be used in the subsequent analysis.

Please note that you can change the descriptor of the used layers (see the following figure).

THE NUMBER OF CRITERIA WHICH CAN BE USED IS LIMITED TO 5! The extension informs you if you want to choose more than 5 raster datasets. However a minimum of two rasters is also required for the analysis.

If you are finished with your criteria definitions click on the button.

The following window appears:
Enter your preference values in the preference matrix by double clicking on the corresponding matrix cell or by hitting the <Enter> key. A window will appear asking you to enter a value. When having entered a valid value, the transpose position in the matrix is automatically inverted. Elements on the matrix’ main diagonal will remain unchanged. Entering values higher than the maximum preference value (which is 9) will be reset to 9, values smaller or equal to 0 will be set to one. The ‘pref. table’ button displays a window which shows a ‘legend’ of the preference values.

To calculate the weights push the **calculate...** button. The calculated weights are displayed in the text box which is underneath. The ‘details...’ button lets you view the details of the calculation in terms of the resulting eigenvalues and more. You can change your preference values at any time. Please note however, that when having changed a preference value the weights are not going to be changed automatically. The extension calculates with the former weights as long as the ‘calculate’ button has not been clicked. To actualize your weights the user must therefore again click the ‘calculate’ button!

When you are done with the weight determination you click the ‘Ok’ button. When the ‘create report file’ check box is checked you will be prompted to enter a name for a text file where all the information of the AHP procedure is stored. If you do not want such a file to be created, deselect the checkbox.

ArcMap performs a raster operation and calculates the weighted sum of the previously defined criteria rasters. The result raster is being added to the current ArcMap session. The default name is ‘ahpResultX’ (where X stands for any integer value).

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**Final remarks**

Please note that the use of the provided extension is for non-commercial use only. One is allowed to use and distribute it freely as long as no-one charges money for it.
References

The following list is by far not complete. However, it is to give the interested reader a minimum starting point to get more information about the AHP and its use in land use decision making.


Example (the example files can be found in the example folder)

The suitability of a piece of land with respect to housing construction has to be found out. For reasons of simplicity only three criteria are being considered. The extension ‘ext_ahp’ expects that all relevant criteria are already classified as integer rasters. In the example these are:

**Criterion 1: Thickness of sand** (classified into 9 classes). The higher the thickness of sand the better.

**Criterion 2: Elevation** (The higher, the better)

**Criterion 3: Slope** (the lower the better which means that high slope values receive a low class value)
AHP evaluation

Step 1: call the extension

Step 2: define the rasterized criteria, define new descriptors if desired

Step 3: determine preference values

The given preference values are a numerical way to express the importance or dominance of one criterion over another. Here, the criterion slope is regarded slightly more important than criterion elevation (value 3 in the first row); the thickness of sand has a medium prevalence over slope and elevation (value of 5).

We can see that criterion sand receives by far the highest weight (~70%). Slope gets about 20%, elevation 9.7% (10%).

Note: The consistency ratio ‘CR’ of 0.13 is above Saaty’s recommended threshold of 0.1. However it is only slightly above 0.1 and we therefore accept the result.

Step 4: Push the ‘calculate’ button; the results are displayed in the text box below the ‘calculate’ button.
Step 5: Push the ‘OK’ button. If you want a report file to be created check ‘create report file’ (default); then specify a filename. A new raster is calculated as a weighted sum of the available rasters and added to the current ArcMap session.

Result raster

![Result raster image]

The most suitable locations are shown in green. One can clearly see that the reddish areas coincide well with the areas of low sand thickness (see page 8) which is quite plausible since criterion sand received a weight of about 70%. Therefore we expect a high influence of our classified sand thickness values in our result map.

Example report file

The report file has the following structure:

```
[Criteria & LayerSource (clsfd.)]
recl_slope    Reclass of recl_slope
recl_sand     recl_sand
recl_elevation recl_elevation

[Preference Matrix]
  recl_slope  recl_sand  recl_elevation
recl_slope 1  0.2  3
recl_sand 5  1  5
recl_elevation 0.3333  0.2  1

[*****AHP results*****]
[Eigenvalues]
3.1356
-0.0678
-0.0678

[Eigenvector of largest Eigenvalue]
0.2747
0.9524
0.1321

[criteria weights]
0.2021  (recl_slope)
0.7007  (recl_sand)
0.0972  (recl_elevation)

[consistency ratio CR]
0.1304
(Revision of preference values is recommended if CR > 0.1)
```