Sensitivity analyses in snow avalanche dynamics modeling and implications when modeling extreme events

Christopher P. Borstad and D.M. McClung

Abstract: We present the first snow avalanche dynamics model simulations to start in the middle of the avalanche path at the maximum expected speed for an extreme event. We first present a sensitivity analysis of the dynamics model to the various model inputs. A single-parameter Coulomb-type friction formulation is used in the model. This formulation is supported by various experiments and full-scale observations of avalanche flow that demonstrate a coupling between the shear and normal forces in flowing snow. The dynamics model is shown to be most sensitive to changes in the friction coefficient. We suggest that the precision in the friction coefficient necessary to confidently use a dynamics model to predict runout distances is higher than the current state of knowledge about avalanche resistance mechanisms. This result leads to the new modeling technique that starts numerical simulations at the midpoint of the length of the avalanche path at maximum speed. The Coulomb friction coefficient is chosen to produce a unique speed profile from this new starting point at maximum speed to a state of rest at an empirically pre-determined runout position. The technique reproduces the observed sharp deceleration of avalanche flow in the runout zone.

Key words: avalanche dynamics, extreme events, sensitivity analysis.

Résumé : Nous présentons dans cet article les premières simulations d'un modèle dynamique d'avalanches de neige qui débutent au milieu du chemin de l'avalanche à la vitesse maximale attendue pour un événement extrême. Premièrement, une analyse de sensitivité du modèle dynamique est présentée par rapport à plusieurs intrants du modèle. Le modèle utilise une formulation de friction de type Coulomb à paramètre unique. Cette formulation est supportée par différents essais et observations à l'échelle réelle d'avalanche qui démontrent un couplage entre les forces de cisaillement et les forces normales dans l'écoulement de neige. Le modèle dynamique est le plus sensible aux variations du coefficient de friction. Il est suggéré que la précision nécessaire du coefficient de friction pour utiliser un modèle dynamique pour prédire les distances de parcours soit plus élevée que le niveau des connaissances actuelles sur les mécanismes de résistance aux avalanches. Ces résultats ont amené à la nouvelle technique de modélisation qui commence les simulations numériques au milieu de la longueur du trajet de l'avalanche à vitesse maximale. Le coefficient de friction de Coulomb est choisi de façon à produire un profil de vitesse unique à partir de ce nouveau point de départ à vitesse maximale jusqu'à l'état de repos à une position prédéterminée empiriquement du parcours. La technique permet de reproduire la décélération marquée de l'écoulement de l'avalanche observée dans la zone de parcours.

Mots-clés : dynamique d'avalanche, événements extrêmes, analyse de sensibilité.

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Introduction

Snow avalanches are a frequent winter natural hazard in alpine countries. Slab avalanches occur after the propagation of fractures in cohesive snow. The result is a large initial volume of flowing snow that accelerates rapidly in the starting zone of an avalanche path. The most destructive flowing avalanches travel very fast and carry a high speed far into the valley bottom where they can destroy mature timber, structures, vehicles or trains. As opposed to wet avalanches, dry flowing avalanches have the highest speeds and peak impact pressures (McClung and Schaerer 1985) and are therefore of most interest to land-use planners involved with avalanche terrain.

Land-use planning and hazard mapping in mountainous avalanche terrain involve making decisions with the aim of minimizing the loss of life, property, and economic activity due to avalanches. The design event typically has a return period of 30 to 100 years or more. Nearly all cases of practical concern involve the avalanche runout zone — typically a valley bottom. Seldom is it necessary to make use of flow calculations in the top half of an avalanche path.

The high frequency of snow slab avalanches, relative to other geophysical natural hazards, allows empirical (statistical) analysis of avalanche behaviour for some planning decisions based on field data. The prediction of an extreme avalanche frontal stopping position, for example, can be made using regional runout data and extreme-value statistics

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(McClung 2001). An advantage of empirical runout prediction is the relative ease with which the uncertainty in the calculations can be quantified.

The design of avalanche defense structures (e.g., deflecting mounds, dams or walls) requires a longitudinal profile of bulk or frontal avalanche speed in the runout zone. These speed calculations must be supplied by a dynamics model. Combined with some prediction of flow density, depth, and length, a longitudinal speed profile allows a calculation of the impact pressure (and thus destructive potential) of the design event as a function of position in the runout zone.

A dynamics model requires the specification of one or more friction parameters. Some characterization of the initial flow geometry is typically necessary in the form of the flow (or fracture) depth, length, and width, as well as the path profile. More complex models may require the specification of additional physical parameters; for example, to account for mass entrainment (e.g, Hungr 1995; Sovilla et al. 2006), internal granular deformation (e.g, Norem et al. 1987; Christen et al. 2002; Sovilla et al. 2006), mass exchange between the dense flowing core and the turbulent powder cloud (e.g., Issler 1998) or variations in the internal pressure state in the flow (e.g., Bartelt et al. 1999; Sartoris and Bartelt 2000). Assigning confidence levels to a dynamics model output requires an analysis of the sensitivity of the output to the model inputs or other model constraints. Unfortunately, sensitivity analyses are rarely reported in avalanche dynamics modeling. Many dynamics models are "verified" with back-calculations of observed events. However, multiple-parameter models have nonunique solutions. For example, Perla et al. (1980) showed that an infinite number of parameter combinations in even a two-parameter model can reproduce a given runout position.

The purpose of this paper is to demonstrate the sensitivity of a particular dynamics model ("dynamic analysis" (DAN) (Hungr 1995)) to the friction coefficient and initial and boundary conditions, such as flow depth, length, and changes in flow width. The analysis is presented for a single-parameter Coulomb-type frictional formulation; the choice of this particular formulation was motivated by the desire for model simplicity and ease of testing model sensitivity to changes in the friction value. We show that the dynamics model output is more sensitive to changes in the friction coefficient than any of the initial or boundary conditions. The implication of such a parameter sensitivity, coupled with the uncertainty about avalanche dynamics, is that the model output is not very robust with respect to the uncertainty in the inputs.

As a response to this excessive sensitivity of model output to uncertain inputs, a new modeling technique is presented. Recorded avalanche speed data are used to supply a dynamics model with an initial, maximum speed in the middle of an avalanche path. This speed constraint is coupled with an independent empirical or field-based prediction of the runout position. The maximum speed and runout distance predictions are used to constrain the selection of the single, bulk friction coefficient in the dynamics model. This allows the calculation of longitudinal speed profiles in the runout zone that are more robust with respect to the model inputs than calculations in the absence of speed and runout distance constraints.

Single-parameter dynamics modeling

In practice, most avalanche dynamics modeling is applied to land-use planning applications. In such applications, the purpose of a dynamics model is to provide outputs, such as runout distance, flow speed, and flow depth, with some level of confidence. It is not necessary in this context to accurately represent the physics of avalanche flow, as desirable as this may be.

A single-parameter friction model is advantageous for a land-use planning application, as it allows easy quantification of uncertainty in the model output based on a sensitivity analysis of the friction coefficient. In a multiple-parameter flow model, the coupled dependence of model parameters makes it more difficult to isolate the sensitivity of model output to individual inputs. In an analytical model, a coupled relationship between the different parameters can be expressed mathematically, as demonstrated for a Voellmy-type one-dimensional model by McClung and Schaerer (1983). In two or three dimensions or when additional parameters are added to a model to account for second-order physical processes, singling out model sensitivity is more difficult.

The case for single-parameter modeling, though seemingly simplistic, has support in the literature. Dent et al. (1998) measured avalanche velocity profiles at a full-scale experimental site and found that most of the shearing is concentrated in a thin layer at the base of an avalanche. Additionally, the ratio of shear to normal stress (S/N) was found to be constant over much of the avalanche. Platzer et al. (2007) measured avalanche S/N ratios in a chute and found that basal shearing without any velocity dependence constituted the majority of avalanche resistance. Tiefenbacher and Kern (2004) observed a thin basal shear layer using the same experimental chute.

McClung (1990), and later Ancey and Meunier (2004), calculated bulk friction coefficients from measured avalanche speeds and runout distances reported by Gubler et al. (1986). The bulk resistance was often nonlinear and three different "regimes" of flow were characterized (Ancey and Meunier 2004). However, in both studies a simple center-of-mass dynamics model using a single, Coulomb-type friction coefficient was able to reproduce observed runout distances and flow speeds. This bulk friction term accounts for all physical resistive processes in the avalanche rather than for any single physical mechanism in the flow. In other words, it is possible to lump all resistive mechanisms into a single bulk friction term and still reproduce the observed avalanche behaviour relevant to planning applications.

The approach presented in this paper involves using a single friction parameter, in the form of a Coulomb sliding friction coefficient, to model extreme avalanches. Extreme events, defined here as avalanches with return periods on the order of 100 years, are necessarily characterized by minimum resistance and maximum speed for a given path. The dynamics model DAN developed by Hungr (1995) is used for the calculations and a sensitivity analysis of model output to various inputs illustrates the need for the new, data-constrained modeling approach.

In the new approach, the flow of an avalanche is modeled from maximum speed in the middle of the avalanche path to zero speed at the final resting position in the runout zone.

The maximum speed is a model input, provided by an envelope curve drawn over measured maximum avalanche speeds scaled with the path length. The runout distance is first estimated by an empirical (statistical) method based on measurements of extreme runout distances in the mountain range of interest.

Given these two constraints, the selection of the single, Coulomb-type friction coefficient is made to fit the speed profile through the two points characterized by maximum speed v_{max} at some mid-path position S_{mid} to a speed $v = 0$ at the frontal stopping position S_{runout}.

Dynamics model description

The numerical model used for this analysis ("DAN," Hungr 1995) was developed for the unsteady flow of earth masses. Though primarily intended for landslides, DAN allows the selection of a variety of material rheologies (e.g., Voellmy, Bingham or frictional) for the flow mass of interest. The model employs a quasi-three-dimensional discretization of the flow mass, with the flow depth and longitudinal length as degrees of freedom.

The flow mass is divided into a variable number of flow blocks, as in Fig. 1. The mass blocks are indexed $i = 1$ to $n - 1$ for the application of the continuity equation. Indexing starts at the upslope boundary of the flow and increments in the downslope direction. Each mass block is bounded longitudinally by two flow cross sections with infinitesimal thickness, oriented normal to the slope. These cross sections are indexed $j = 1$ to n for the iterative application of the momentum equation.

The flow path is approximated as a straight rectangular channel without any curved channel bends. The model flow width is defined as the channel width at the surface of the flow. The width can vary along the path and, in DAN, the flow width must be input by the user based on knowledge of channel confinement, vegetative clues or historical experience with the path of interest. The flow depth, following the theory of open channel flow, is interpreted as the hydraulic depth, defined as the cross-sectional flow area divided by the surface flow width. This allows approximate characterization of irregular channel cross sections in this rectangular channel model. In practice, avalanche path cross sections range from convex alluvial fans to narrow, confined channels.

DAN contains a Lagrangian numerical scheme, such that the curvilinear coordinate system moves downslope with the flow. This approach is believed to be more appropriate for unsteady flow equations than a fixed-grid, Eulerian numerical scheme (Potter 1972). A first-order finite difference discretization of the equations of motion allows for easy numerical computation of position and velocity updates for each block boundary. Figure 2 shows the geometry of a mass block and the adjacent cross sections, which are considered to have infinitesimal thickness.

Newton's second law acting on a cross section is expressed as

$$[1] \qquad \frac{\mathrm{d}}{\mathrm{d}t}(M_i v_i) = F_i$$

where t is time, M_i is the mass of the cross section i, and v_i is its velocity. F_i can be expressed as a sum of forces

Fig. 1. Profile view of the flow discretization in the DAN model. Mass flow blocks are bounded by cross sections oriented normal to the slope. The mass blocks are indexed $j = 1$ to $n - 1$ and the bounding cross sections are indexed $i = 1$ to n.

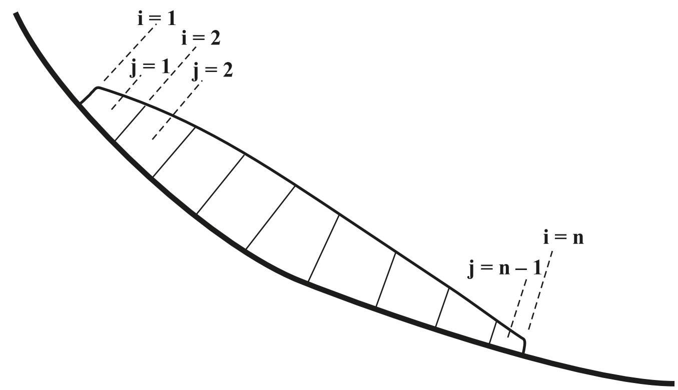
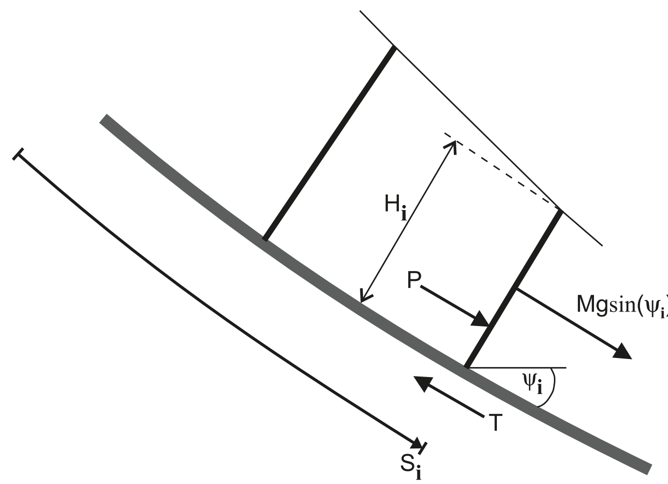


Fig. 2. Single mass block bounded by two cross sections in the DAN model. H_i, section height; $Mg\sin(\psi_i)$, gravitational driving force; P, internal pressure; S_i, position coordinate; T, flow resistance; ψ_i, slope angle.



$$[2] \qquad F_i = M_i g \sin\psi_i + P_i - T_i$$

where g is gravitational acceleration, ψ_i is the slope angle, P_i is the internal pressure, and T_i is the basal resistance. The mass of cross section i is

$$[3] \qquad M_i = \rho H_i B_i \,\mathrm{d}S$$

where ρ is the material density, H_i and B_i are cross-section height and width, respectively, and dS is the infinitesimal thickness. The internal pressure term is written as

$$[4] \qquad P_i = -kM_i g \frac{\mathrm{d}H}{\mathrm{d}S}\left(1 + \frac{a_c}{g}\right)\cos\psi_i$$

where k is the longitudinal pressure coefficient and dH/dS is the longitudinal gradient in flow depth. The term $a_c = v_i^2/R$ is the apparent centrifugal acceleration that arises in the noninertial Lagrangian reference frame for a curved path with a vertical radius of curvature R.

In Rankine earth-pressure theory, the pressure coefficient k takes values in the range 0.2 for a static, "extending" (active) mass to 5.0 for a compressing (passive) mass. These static states are analogously associated with an extending and compressing flow strain in DAN, as is common practice in avalanche flow modeling. The model DAN can modify the internal pressure state, varying k at each time step based on the state of longitudinal strain in the flow. For a description of the strain calculation, and the associated update in k, refer to Hungr (1995).

It should be noted that this common internal pressure formulation is purely speculative and is not based on any known experimental avalanche data, in spite of theoretical attempts to apply it (Bartelt et al. 1999). In any case, the value of k has little influence on either the runout distance or the frontal speed in DAN (Borstad 2005). Given the uncertainty in the internal pressure state of flowing snow, and for additional numerical simplicity, k was held fixed at the passive value k_p = 2.5 for the following analysis, as compressive flow strain is expected in the runout zone for most avalanche paths (McClung and Mears 1995).

The basal resistance term T can take a variety of forms in DAN. For dense snow avalanche flow, if a bulk Coulomb-like sliding friction is assumed as the dominant resistance parameter, the basal resistance takes the form

$$[5] \qquad T_i = \mu M_i (g \cos\psi_i + a_c)$$

where μ is the Coulomb (or bulk) friction coefficient, which can vary along the length of the slope, and a_c is again the centrifugal term. This equation expresses the observed coupling between the shear and normal forces in flowing snow (e.g., Dent et al. 1998; Platzer et al. 2007) and other granular flows (e.g., Bagnold 1954; Louge and Keast 2001).

Substitution of eqs. [2]–[5] into a first-order discretization of eq. [1] leads to a solution for the velocity of a cross section at a new time step

$$[6] \qquad v_i = v_i' + \frac{g(F_i \, \Delta t - p)}{\rho H_i B_i \, \mathrm{d}S}$$

where Δt is the time-step size and a prime symbol (′) indicates the value at the previous time step. The term p is a momentum flux term to account for the entrainment or deposition of material. Neither process was considered for this study, and p was set to zero.

Once the new velocities have been computed for each cross section at the new time step from eq. [6], the displacements S_i (in m) are computed as

$$[7] \qquad S_i = S_i' + \frac{\Delta t}{2}\left(v_i + v_i'\right)$$

The continuity equation allows the calculation of the average depth of a mass block. If mass entrainment and deposition are neglected, the mass of each block is constant and the block depth is calculated as

$$[8] \qquad h_j = \frac{2V_j}{(S_{i+1} - S_i)(B_{i+1} + B_i)}$$

where V_j is the volume of the mass block. The new height of each cross section is then calculated as the average of adjacent mass blocks

$$[9] \qquad H_i = \frac{h_{j-1} + h_j}{2}$$

The front and tail of the flow are tapered off such that the heights of the outer cross sections are each set at half the height of the adjacent mass block

$$[10] \qquad H_1 = \frac{h_1}{2}, \qquad H_n = \frac{h_{n-1}}{2}$$

At this point the solution is complete for the current time step, unless a variable internal pressure calculation is made (Hungr 1995). The solution iterates until all blocks have come to rest.

DAN sensitivity analysis on a smooth parabolic path

A sensitivity analysis was performed to illustrate the role of the different physical parameters used in DAN and their influence on speed and runout. A hypothetical parabolic path, defined by the equation $y = 0.0003x^2 - 1.2x + 1200$, was considered. A vertical fall of 1200 m is close to that of many large avalanche paths that threaten facilities with potentially destructive extreme events.

The representation of an avalanche path as a second-order polynomial allows the determination of unique terrain parameters for use in comparative statistical prediction of extreme runout. This is due to the smoothly changing nature of a parabolic approximation of three-dimensional terrain. Third-order or higher polynomial representation may involve changes in curvature and introduce ambiguity in the location of terrain parameters such as the β-point, defined as the point where the slope first decreases to 10° (Lied and Bakkehoi 1980). Though refined dynamics modeling allows more realistic characterization of actual avalanche paths, a parabolic path is a good starting point from which to explore and compare the behaviour of any dynamics model.

In two- or three-dimensional dynamics modeling, it can be difficult to isolate the sensitivity of individual parameters. This is often due to the coupled dependence of two or more parameters, as demonstrated for the Voellmy friction formulation by McClung and Schaerer (1983). In a discrete flow model, for example, the flow length, width, and depth may have a complicated interdependence. The analysis that follows demonstrates some of the stronger sensitivities encountered in avalanche dynamics modeling that can be approximately singled out. For the modeling here, entrainment and deposition of flow mass were neglected, and a single bulk friction coefficient provided the flow resistance. The model inputs in this scenario were the friction coefficient, initial flow length, initial flow depth, and flow width.

Sensitivity to flow length and depth

The initial flow length is typically equated with the downslope slab fracture dimensions, and the depth with the initial slab thickness. Figure 3 shows the influence of changing flow depth on the speed profile and runout distance, given an initial flow length of 100 m. In this case, both the maximum speed and runout distance are not very sensitive to a factor of 10 change in the flow depth from 0.5 to 5 m. The Coulomb (or bulk) friction coefficient and the flow width were held constant for each simulation.

Given a longer initial flow length over the same range of flow depths, however, the results are quite different. Figure 4 shows speed profiles for a 500 m initial flow length. The maximum speed reached in these two cases, around 50 m/s, is much lower than for the flows with a shorter initial length (Fig. 3), which reached maximum speeds greater than 60 m/s. The runout distance is greater for the longer initial flow length despite these lower speeds, however, and the maximum speed and runout distance are much more sensitive to the initial flow depth.

A striking feature of Fig. 4 is the small peak in the speed of the flow toward the end of the path. This brief acceleration occurs as the slope angle is decreasing, so it cannot be reproduced by any one-dimensional model, assuming the friction is held constant. The reason for this feature is that as the flow front begins to decelerate upon reaching lower-slope angles, the tail of the flow is still traveling relatively faster on steeper terrain upslope. As an assumption in DAN is that the flow is incompressible, the tail of the flow transfers momentum to the front as it catches up in the runout zone. This characteristic was observed in DAN for many simulated avalanche paths if the flow length was longer than a critical threshold, which was dependent on the path. Anecdotal observations by avalanche technicians suggest that similar behaviour is observed in actual avalanches (B. McMahon, personal communication, 2007). Additionally, speed features of this nature are present in some of the data reported (Madergrond path) by Gubler et al. (1986) and could perhaps be explained by this phenomenon observed in DAN.

Width sensitivity

To investigate the effect of changing width over the course of a given path, the runout zone width of the same parabolic path was altered. The width of the track was held constant until the slope angle decreased to 16° at x = 1500 m. From this point, the width was linearly increased or decreased until the slope angle reached 0° at x = 2000 m, after which the width was held constant. This served to replicate different measures of lateral spreading or constriction in the runout zone. Figure 5 shows a nondimensional runout ratio (McClung and Mears 1995) versus the ratio of runout zone width to track width. The runout ratio is defined as $\Delta x/X_\beta$, where Δx is the horizontal distance between the β-point where the slope angle first decreases to 10° and the frontal stopping position of the slide, and X_β is the horizontal distance between the β-point and the start position of the slide. When compared with runout for a constant-width channel, the runout ratio (and therefore runout distance) decreases as the flow spreading increases in the runout zone, as may be expected.

The DAN model requires that flow width be input by the user. Therefore, the most conservative runout and speed calculations arise from holding the flow width constant in the runout zone, unless terrain confinement for the path in question suggests otherwise. Flows often spread laterally in the valley bottom, leading to shorter runout distances than if the flow width remained constant. Few avalanche paths have constrictions that would decrease the flow width in the runout zone. In these cases, however, the path geometry typically makes it easy to specify the width in a model. In any case, Fig. 5 shows that the model results are very sensitive to a change in the flow width.

Fig. 3. Speed profiles for different initial flow depths, given a 100 m initial flow length.

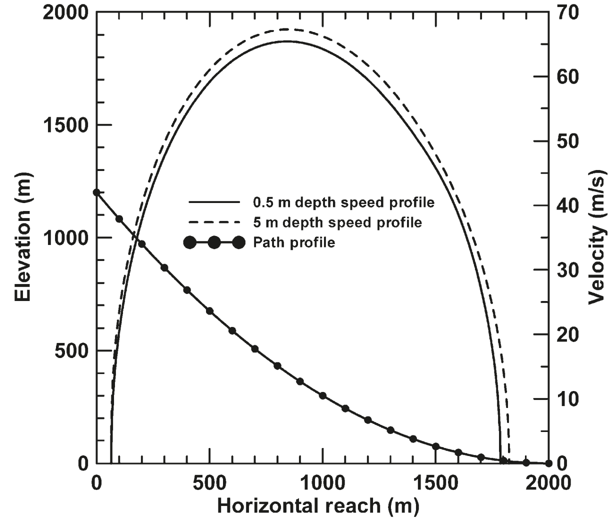
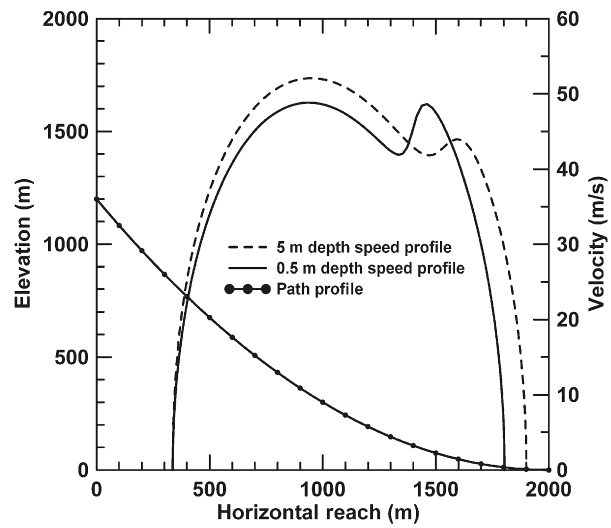


Fig. 4. Speed profiles for different initial flow depths, given a 500 m initial flow length. All other parameters were held the same as in Fig. 3.



In a model like DAN, the flow width in a laterally unconfined runout zone could be specified based on data from avalanche deposits. As in Fig. 5, this could take the form of measuring ratios of the flow width in the track to the width of the avalanche deposit. This would provide a constraint that may yield more realistic runout results between the conservative limit assuming constant width (no spreading) and the opposite limit obtained by speculating that the flow runs the full width of the runout zone (maximum spreading).

A model that predicts flow spreading (unlike DAN) would have to make some calculation about the state of lateral pressure in the flow. Similar to a longitudinal pressure formulation (e.g., eq. [4]), this would currently require a somewhat speculative numerical formulation as the internal pressure state of flowing snow is unknown. This would lead to additional model parameterization and further difficulty in quantifying the confidence or uncertainty in extreme avalanche predictions.

Fig. 5. Nondimensional runout ratio (as defined in McClung and Mears (1995)) versus ratio of runout zone width to track width. As the flow width increases in the runout zone, the runout distance decreases, and vice versa.

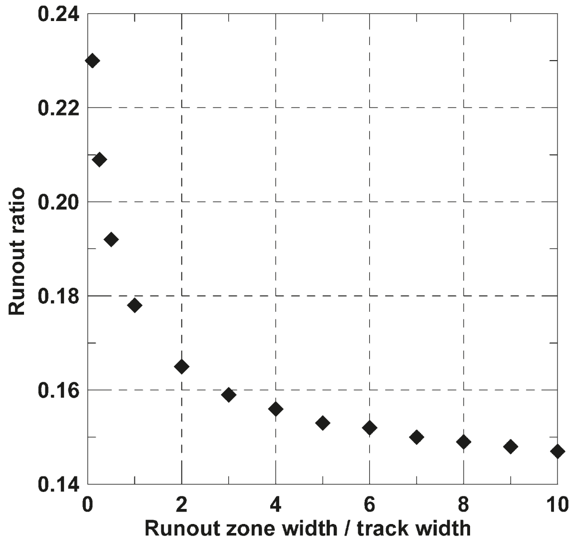
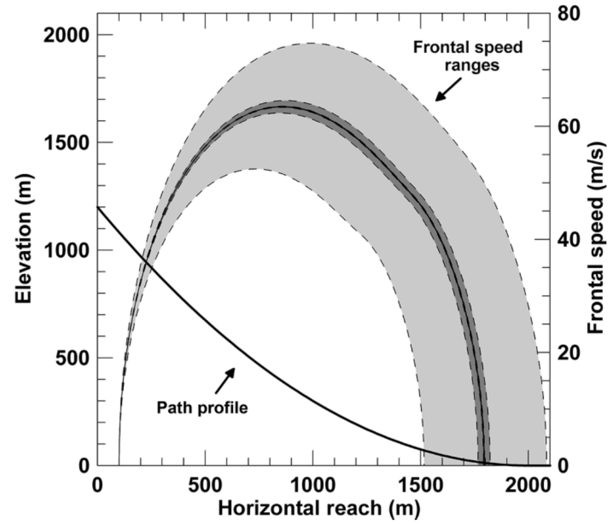


Fig. 6. Sensitivity of DAN output to different variations in the friction coefficient μ. The thick line is the path profile. The thin solid line represents the speed profile for μ = 0.6. The dark gray region represents μ ± 0.01 and the light gray region μ ± 0.1.



Friction sensitivity

The most fundamental parameters in any avalanche dynamics model are the friction coefficients. Sensitivity analyses of runout distance to the friction parameter(s) are typically not reported, however. Such an analysis would shed light on the precision that would be necessary in a forecast of the friction for an extreme event.

In Swiss avalanche dynamics models, the friction coefficients for an avalanche of a given return period are specified in a set of guidelines (Salm et al. 1990). These parameters are typically the pair (μ, ξ) representing a Coulomb-type friction and a turbulent friction proportional to the flow speed squared, respectively. In these Swiss guidelines, these parameters are specified to at most two significant figures. In DAN, however, the runout distance is sensitive to the third significant digit of μ (eq. [5]) for most avalanche paths. Ancey and Meunier (2004) analyzed the runout distance for the Arabba avalanche path in Switzerland, with similar results. To reproduce a given runout position, three significant figures in the bulk friction μ were needed.

Figure 6 shows DAN calculations for the parabolic path for different order-of-magnitude uncertainties in the friction coefficient μ. Changing the third digit of μ, which would correspond to less than 1% uncertainty in the forecast value of the flow friction, leads to a difference in runout distance of around 10 m. The line thickness of the central speed profile in Fig. 6 represents this level of model sensitivity. If the uncertainty in avalanche friction was in the second digit of μ, or on the order of a few percent, then the range in model output is much narrower. In this scenario (changes in μ around 1%–2%), the uncertainty in the calculated runout distance approaches 100 m. At a given position in the runout zone, the uncertainty in speed as the flow decelerates is on the order of 10 m/s.

If the uncertainty in avalanche friction is in the first digit of μ, or on the order of 10% or more, then the model output varies widely. The runout distance in this scenario, represented by the light gray shaded area in Fig. 6, varies by over 500 m. In the runout zone at x = 1800 m, the uncertainty in the flow speed ranges from 0 to about 50 m/s.

This analysis suggests that it is unrealistic to use a dynamics model to estimate the runout distance. The required precision in μ is not available, as indicated by the large range in reported values of μ in the literature (e.g., Dent et al. 1998; Ancey and Meunier 2004; Tiefenbacher and Kern 2004; Platzer et al. 2007). For multiple parameter models, the procedure is even more speculative. Assigning confidence to the output of a model using some kind of sensitivity analysis becomes increasingly difficult as additional parameters are incorporated.

New modeling technique

Even in the minimally parameterized scenario discussed above, the sensitivity of the model results to the inputs was often quite strong. This calls into question the use of more complex avalanche dynamics models with governing equations containing speculative numerical and physical parameters, especially from a land-use planning perspective. It is desirable academically to attempt to understand and reproduce as closely as possible the complex dynamics of an avalanche. However, when making decisions that affect the safety of people and structures in avalanche terrain, a simpler model with some confidence assigned to the results seems more appropriate.

Given the high sensitivity of runout distance to the input friction coefficient (Fig. 6), and the uncertainty surrounding the resistive forces in avalanche flow, a dynamics model is not the best means for predicting runout distances. Back-calculations of numerous observed events may lead to some constraints on the input parameters, but it would be hard to imagine a set of inputs confidently calibrated to better than 10% uncertainty for a complex geophysical process such as a snow avalanche.

To address this sort of problem, McClung (1990) suggests first predicting a runout distance using an empirical method, calibrated with regional field data. This serves to constrain the selection of the friction coefficient for a dynamics model — the friction is adjusted to reproduce the statistically predicted runout position. The most important output of the dynamics model is then the speed of the avalanche as a function of position along the slope as the avalanche approaches the predicted runout position. These speed calculations, together with estimates of the flow depth and density, give a prediction of the impact pressure and thereby, the destructive potential of an extreme event. Alternatively, the speed estimates can be used to design avalanche runout zone defenses.

Building on this procedure, the DAN model was used to calculate the speed profiles of avalanches, given an empirically specified runout position. A further constraint was provided by a wealth of data on maximum avalanche speeds. McClung (1990) reported maximum measured avalanche speed data scaled with the path length for avalanches in Canada, Switzerland, and Norway. A conservative upper-limit envelope for maximum avalanche speed can be drawn over the data. This envelope is represented by the equation

$$[11] \qquad v_{\max} = 1.5\sqrt{S}$$

where $v_{\max}$ is the maximum flow speed and S is the curvilinear path length.

This speed data can be used to constrain the selection of sensitive input parameters, such as flow depth and length, which could otherwise give rise to quite different maximum speed calculations (as in Figs. 3 and 4). The upper-limit envelope defines the extreme avalanche for dynamics modeling by specifying an upper bound on observed speeds. Consequently, calibrating the model using this envelope restricts the modeling domain to extreme events only. High-frequency events, which necessarily have lower speeds and thus higher friction, cannot be modeled using this approach.

DAN was designed, as are most mass-flow models, to simulate the flow from initiation in a starting zone to a state of rest at some position downslope. The initial conditions for the calculations are thus $v = 0$ at some initial position S_o at $t = 0$. The simulation continues until the flow decelerates to rest and again $v = 0$ at some position $S = S_{\text{final}}$ at time $t = t_{\text{final}}$.

Given a priori predictions of runout distance and maximum speed, however, the domain of avalanche dynamics modeling can be restricted to the calculation of speed profiles in the lower reaches of avalanche paths. Seldom are speed calculations necessary in the upper half of an avalanche path, where risk is extremely high and risk-uncertainty is low. Most land-use planning applications in avalanche terrain involve a valley bottom. For this reason, the proposed modeling alternative starts with restricting the domain of interest to only the lower half of an avalanche path.

Rather than start the flow from rest, the flow is given an initial nonzero speed at the midpoint of the path length, such that the a priori maximum speed is reached. Additionally, the bulk friction coefficient is adjusted such that the a priori runout distance is matched. In this way, the fracture dimensions no longer need to be forecast for an extreme event. The model inputs are still the flow length, depth, width, and friction coefficient, but now the simulations are started from an initial nonzero speed rather than from rest.

The flow width at the midpoint of the length of many avalanche paths is constrained topographically and is relatively easy to specify as a model input compared with predicting the fracture dimensions in a large open start zone. Flow depths can be inferred from impact pressure measurements, which indicate that the most dense part of the flow has a typical depth of 2 m or less (Schaerer and Salway 1980; McClung and Schaerer 1985; Schaer and Issler 2001). The remaining geometric input, the flow length, can be selected to reproduce the expected avalanche volume according to the appropriate avalanche size classification (e.g., McClung and Schaerer 1981) or the estimated yield of snow for the design event (e.g., Schaerer 1988).

In the runout zone, the width of the flow often needs to be adjusted by trial and error, as an extreme event will not necessarily run the full width of the existing path. The less the avalanche spreads, the further it will travel (Fig. 5), therefore the most conservative speed estimates will arise from maintaining constant-width flow. The flow width can be easily adjusted in DAN to reproduce the given runout position while at the same time reaching the given maximum speed.

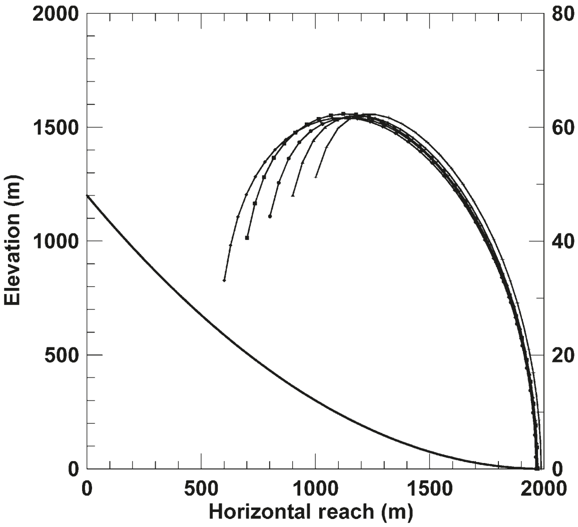
Mass entrainment as the flow decelerates is neglected in this modeling procedure. Entrainment involves both the dislodging of stationary mass and the acceleration of that mass from rest, both of which are expected to consume energy and increase the flow resistance. This modeling procedure is limited to extreme events that necessarily involve minimum resistance.

Starting position sensitivity

Figure 7 shows, for the hypothetical parabolic path, that the frontal starting position has little influence on the speed profile if the runout position and maximum speed are fixed. The brief period of acceleration presented in Fig. 7 is due to the longitudinal spreading of the flow after it is set in motion. Due to this spreading or thinning of the flow, it was not possible to set the initial speed of the flow exactly at the given maximum speed and strictly model the deceleration of the flow, as was originally desired. This would be possible in a one-dimensional or lumped-mass model, but in DAN this brief longitudinal thinning was unavoidable. Each flow-block cross section received an individual initial speed at the first time step. The distribution of the block speeds from the front to the rear of the flow was altered in a variety of ways, and in every case the flow mass thinned longitudinally and caused a brief frontal acceleration. This feature was less prominent when modeling actual avalanche paths, which typically have irregular flow profiles (Figs. 8 and 9) compared with the smooth nature of the parabolic path considered in Fig. 7.

In the end, for simplicity, each indexed flow cross section was given the same initial speed. By a process of trial and error, the starting position was varied in order for the maximum speed to occur at the point on the path where the slope angle ψ first decreased to $\psi = \tan^{-1}(\mu)$. This corresponds to the point where, in a one-dimensional model with a single bulk friction coefficient, the gravitational driving force is exactly balanced by the resisting forces, and acceleration

Fig. 7. Speed profiles for different starting positions given a maximum speed defined by the envelope curve in eq. [11] and a hypothetical empirically predicted runout distance. Both μ and the maximum speed were the same for each calculation.



ceases. For the parabolic path in Fig. 7, the maximum speed for every starting position occurred very near the point on the slope described by $\psi = \tan^{-1}(\mu)$.

Figure 8 shows a similar analysis for the Madergrond avalanche path in Switzerland. This avalanche path is slightly more sensitive to changes in the model starting position. In this case, the maximum speed occurs at a different point for each simulation, although the deceleration profiles far into the runout zone are very similar. The choice of the proper starting point can be constrained, in absence of field-based judgements, so that the peak speed occurs where $\psi = \tan^{-1}(\mu)$. In practice, however, such decisions are best conditioned by field visits, local experience, and topography.

Comparison of model output with observed events

This new modeling method was compared against a measured runout position and avalanche speed profile. In this case, rather than selecting a maximum speed from eq. [11], the measured maximum speed was used. Additionally, the location where this maximum speed occurred provided a further constraint on the back-calculation. As discussed above, in practice the initial model speed was set at a slightly lower value than the measured maximum speed due to the small acceleration caused by the longitudinal thinning once the flow was set in motion. Figure 9 demonstrates the output of the DAN model, compared against measured speed data and a single-parameter model output from McClung (1990). Both models show good agreement with the avalanche deceleration and runout position. The model of McClung (1990) slightly overestimates the speed in the middle of the path when only a single friction parameter is used. The addition of a speed-dependent term to account for top surface drag reduced the speed in the middle of the path, in better agreement with the measured speed, but at the expense of additional parameterization. The single-parameter method presented here avoids this difficulty by starting in the central part of the path using measured speed data that implicitly include surface drag. The observed sharp deceleration in the runout zone is reproduced for both single-parameter models. This result is critical for any decisions, for example, that depend on impact pressures derived from model speeds. The advantage of the present approach, in a predictive application, is the presence of two model constraints (maximum speed and runout distance) based on field data rather than the one constraint (runout distance) in McClung (1990).

Fig. 8. Speed profiles for different starting positions for the Madergrond avalanche path. Arrows indicate the position on the slope from which each calculation began. Both μ and the maximum speed were the same for each calculation. Changes in the model starting position had little effect on the speed profile and runout distance.

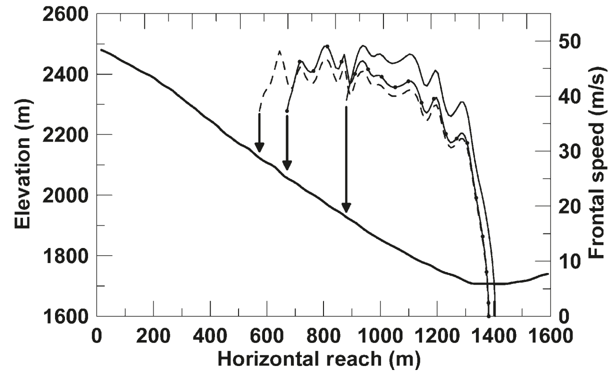
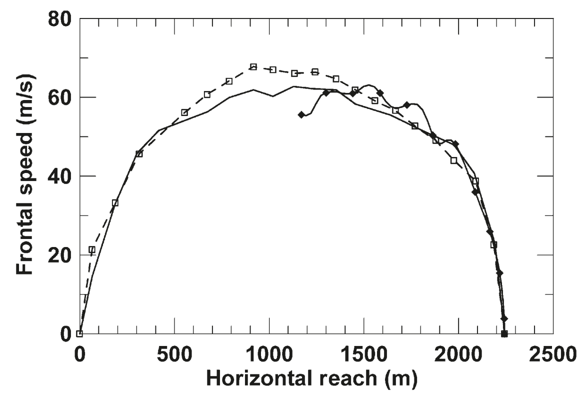


Fig. 9. Speed profiles for the Aulta avalanche path in Switzerland. (—), measured speeds; (□), one-dimensional model of McClung (1990); (♦), DAN using the current approach (density of model points is greater than density of diamond symbols). Speed data modified from Gubler et al. (1986).



Summary

The flow speed and runout distance are the most important outputs of a dynamics model in a land-use planning application. The safety of people and structures in avalanche terrain depend on the confident calculation of avalanche speed for the design event, especially as the flow decelerates.

Recent advances in avalanche dynamics observation and modeling have indicated that a single-parameter frictional formulation is able to adequately capture the bulk behaviour of avalanche flow. Nevertheless, the sensitivity of a model output to even a single parameter is quite high. This is an unfortunate result considering the widespread use of dynamics models, most of which have multiple uncertain parameters. For this reason, additional constraints were sought to manage the sensitivity of a model output to uncertain inputs.

Using the traditional modeling approach and a single friction coefficient, the preceding analysis demonstrates that the precision in the friction coefficient needed to reproduce an observed runout position is very high. This finding has crucial implications for using dynamics models to predict the runout distance of extreme events. In such applications, the friction coefficient would have to be forecast with a precision that is not likely possible given the uncertainty in our knowledge of avalanche dynamics and the variability in the properties of flowing snow. Therefore, empirical methods of calculating the avalanche stopping position should take precedence over the calculations from dynamics models for extreme events.

Compounding the friction uncertainty is the sensitivity of the model output to initial flow geometry and flow width in the runout zone. Changes in flow width are second in importance to the friction coefficient in influencing the flow speed and runout distance in DAN. As with the friction coefficient, the initial and boundary conditions of an extreme event would be difficult to forecast with great confidence for most avalanche paths. Additionally, individual model uncertainties are not independent and lead to difficulty in quantifying the uncertainty in final calculations of flow speed and runout distance. This is the cost of moving from simple one-dimensional models to more complex two- and three-dimensional models.

For these reasons, additional model constraints based on field data were pursued. Extreme runout distance data from the mountain range of interest can be used to supply an a priori runout distance calculation, with a quantified uncertainty, using an empirical (statistical) method. Avalanche speed data can be used to supply an a priori maximum flow speed. Given these two constraints, the new modeling technique restricts the domain of interest to the lower half of the avalanche path. An initial, nonzero speed is given to the flow in the middle of the path such that the a priori maximum speed is reached. From this point, the model calculations proceed until the a priori runout distance is reached. The longitudinal speed profile is then well constrained by data that implicitly contain information (such as entrainment) on extreme events.

The DAN model applied using this technique is able to reproduce the sharp deceleration that is observed in avalanche runout zones. The single bulk friction coefficient implicitly accounts for all resistive processes, both internal and at the boundaries. Additionally, the model provides information about the longitudinal profile of the flow mass and deposit.

It should be stressed again that this modeling technique is limited to extreme events only. More frequent events with shorter return periods are necessarily characterized by higher bulk flow resistance. The comprehensive approach presented here could be systematically incorporated into a distribution package for application in the avalanche engineering community. It should be noted, however, that prescriptive modeling approaches are incomplete unless conditioned by proper engineering experience and judgement.

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